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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2014/2015

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS II
COURSE CODE	:	BFC 14003
PROGRAMME	:	BACHELOR OF CIVIL ENGINEERING WITH HONOURS
EXAMINATION DATE	:	JUNE 2015 / JULY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS

THIS QUESTION PAPER CONSISTS OF SIX **(6)** PAGES

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Q1 (a) Solve the differential equation $e^x \frac{dy}{dx} + 2e^x y = 1$, given that $y = 2$ when $x = 0$.

(9 marks)

(b) The motion of an object is governed by the equation $\frac{dv}{dt} = g - kv$, where v is the velocity at time t, g is the gravity and k is a constant.

(i) Find the velocity v by assuming that the object starts from rest.

(ii) Deduce that after a long period of time, the object will move with a constant velocity $\frac{g}{k}$.

(11 marks)

Q2 (a) Given a non-homogenous second order differential equation

$$y'' - y' - 2y = e^{3x}$$

Fine the general solution for the equation by using variation of parameters method.

(10 marks)

(b) A mass weighing 30 kg stretched a spring 1.5 m. Assuming a dumping force that is 9 times the velocity of the mass, determine equation of the motion, if the mass is initially released from a point 0.3 m below the equilibrium position with a upward velocity of 1.5 m/s and m = 1.

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

(10 marks)

Q3 (a) Find the Laplace transform for each of given function

(i) $f(t) = 3e^{-5t} + 5t^3 - 9$

(ii) $g(t) = 4\sinh(2t) + 3\sin(2t)$

(4 marks)

(b) Find

$$\mathcal{L}^{-1}\left\{\frac{3s+5}{s^2 - 5s + 6}\right\}$$

(6 marks)

(c) By using Laplace transform method, solve

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} = e^t cost$$

Given $y(0) = 0, y'(0) = 0$

(10 marks)

Q4 (a) For the following power series determine the interval and radius of convergence.

$$\sum_{n=0}^{\infty} \frac{6^n}{n}(4x-1)^{n-1}$$

(10 marks)

(b) Use one of the Taylor Series derived to determine the Taylor Series for $f(x) = e^{-6x}$ about $x = -4$..

(10 marks)

Q5 (a) A periodic function is defined by

$$f(x) = \begin{cases} 0, & -\pi < x < -\dfrac{\pi}{2}, \\ 4, & -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}, \\ 0, & \dfrac{\pi}{2} < x < \pi, \end{cases}$$

$f(x) = f(x + 2\pi)$.

(a) Sketch the graph of the function over $-2\pi < x < 2\pi$.

(6 marks)

(b) Determine whether the function is even, odd or neither.

(2 marks)

(c) Show that the Fourier series of the function $f(x)$ is

$$2 + \frac{8}{\pi}\sum_{n=1}^{\infty} \frac{\sin\left(\frac{n\pi}{2}\right)}{n}\cos nx$$

(12 marks)

-END OF QUESTION-

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Formulae

Characteristic Equation and General Solution

Case	Roots of the Characteristic Equation	General Solution
1	m_1 and m_2 ; real and distinct	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	$m_1 = m_2 = m$; real and equal	$y = (A + Bx)e^{mx}$
3	$m = \alpha \pm i\beta$; imaginary	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Undetermined Coefficients

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + \cdots + A_1 x + A_0$	$x^r(B_n x^n + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Variation of Parameters

Wronskian	Parameter	Solution
$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$	$u_1 = -\int \frac{y_2 f(x)}{W} dx, \quad u_2 = \int \frac{y_1 f(x)}{W} dx$	$y_p = u_1 y_1 + u_2 y_2$

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Laplace Transforms

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
$t^n,\ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
e^{at}	$\frac{1}{s-a}$	$\delta(t-a)$	e^{-as}
$\sin at$	$\frac{a}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s).G(s)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$y(t)$	$Y(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$\dot{y}(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$\ddot{y}(t)$	$s^2Y(s)-sy(0)-\dot{y}(0)$
$t^n f(t),\ n=1,2,3,\ldots$	$(-1)^n\frac{d^nF(s)}{ds^n}$		

Periodic Function for Laplace transform : $\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt, \quad s>0.$

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Fourier Series

Fourier series expansion of periodic function with period $2L$	**Fourier half-range series expansion**
$f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{L}+\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{L}$	$f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{L}+\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{L}$
where	where
$a_0=\frac{1}{L}\int_{-L}^{L}f(x)\,dx$	$a_0=\frac{2}{L}\int_{0}^{L}f(x)\,dx$
$a_n=\frac{1}{L}\int_{-L}^{L}f(x)\cos\frac{n\pi x}{L}\,dx$	$a_n=\frac{2}{L}\int_{0}^{L}f(x)\cos\frac{n\pi x}{L}\,dx$
$b_n=\frac{1}{L}\int_{-L}^{L}f(x)\sin\frac{n\pi x}{L}\,dx$	$b_n=\frac{2}{L}\int_{0}^{L}f(x)\sin\frac{n\pi x}{L}\,dx$