CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2014/2015

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS 1
COURSE CODE	:	BFC 13903
PROGRAMME	:	BACHELOR OF CIVIL ENGINEERING WITH HONOURS
EXAMINATION DATE	:	JUNE 2015/JULY 2015
DURATION	:	3 HOUR
INSTRUCTION	:	ANSWER ALL QUESTIONS

THIS QUESTION PAPER CONSISTS OF **TEN (10)** PAGES

CONFIDENTIAL

Q1 (a) Determine whether the function below is continuous at $x = 0$. Give reason to your answer.

$$f(x) = \begin{cases} \dfrac{\sin(6x^2)}{\tan(3x^2)} & if\ x \neq 0 \\ 2 & if\ x = 0 \end{cases}$$

(7 marks)

(b) Evaluate the limits of the following expressions:

(i) $\lim\limits_{w\to\infty} \dfrac{0.5w + 10}{7e^{-0.379w}}$

(3 marks)

(ii) $\lim\limits_{x\to4^+} \dfrac{3}{(4-x)^3}$; $\lim\limits_{x\to4^-} \dfrac{3}{(4-x)^3}$; $\lim\limits_{x\to4} \dfrac{3}{(4-x)^3}$

(4 marks)

(c) Evaluate the following limit by using **L' Hospital's Rules**.

(i) $\lim\limits_{x\to1} \dfrac{5x^3 - 6x^2 - 7}{21 - x - 9x^2}$

(3 marks)

(ii) $\lim\limits_{x\to\infty} \dfrac{e^x}{x^4}$

(3 marks)

Q2 (a) **FIGURE Q2** shows a metal cube which is expanding uniformly as it is heated. At time t seconds, the length of each edge of the cube is x cm and the volume of the cube is V m^3. To produce pad foundation, the size of metal cube is used.

(i) Show that $\dfrac{dV}{dx} = 3x^2$

(2 marks)

(ii) Given that the volume, V m^3, increases at a constant rate of 0.048m^3/s.

Find $\dfrac{dx}{dt}$ when x=8

(5 marks)

(iii)Find the rate of increase of the surface area of the cube, in m^2/s, when $x=8$

(5 marks)

(b) By using **First Derivative Test**, determine the relative maximum and minimum of $f(x) = -(x+3)^2 + 1$

(8 marks)

Q3 (a) Integrate the following using **Integration by Partial Fractions** method.

$$\int \frac{x^4+x^3+x^2+1}{x^2+x-2} dx$$

(10 marks)

(b) Evaluate the following integral by using **Integration by Parts** technique.

$$\int (3t+5)\cos(\frac{t}{4})dt$$

(5 marks)

(c) Evaluate $\int \frac{15}{3-2x} dx$ by using **Substitution** technique.

(5 marks)

Q4 (a) Differentiate the following:

(i) $y = sinh^{-1}(\tan x)$

(4 marks)

(ii) $f(t) = 4cos^{-1}(t) - 10tan^{-1}(t)$

(2 marks)

(iii) $f(x) = x^{-1/2}sin^{-1}(x)$

(3 marks)

(b) Evaluate $\int \frac{3dx}{25+16x^2}$ using **Inverse Trigonometric Function**.

(6 marks)

(c) Integrate $\int \frac{dx}{4-x^2} dx$ using **Hyperbolic Function**.

(5 marks)

Q5 (a) Find the radius of curvature of $y = 3x^3 - 2x^2 - x + 10$ at point $x = 5$.

(7 marks)

(b) Determine the length of $y = \ln(\sec x)$ on $0 \leq x \leq {}^{\pi}/_{4}$.

(8 marks)

(c) Determine the area of the region bounded by $y = x + 1$ as the upper boundary and $y = xe^{-2}$ as the lower boundary.

(5 marks)

- END OF QUESTION –

FINAL EXAMINATION

SEMESTER / SESSION: SEM II / 2014/2015 COURSE : 1 BFF/ 2 BFF/ 3 BFF/ 4 BFF

SUBJECT : CIVIL ENGINEERING MATHEMATICS I CODE : BFC13903

Formulae

Trigonometric	Hiperbolic
$\cos^2 x + \sin^2 x = 1$	$\sinh x = \dfrac{e^x - e^{-x}}{2}$
$1 + \tan^2 x = \sec^2 x$	$\cosh x = \dfrac{e^x + e^{-x}}{2}$
$\cot^2 x + 1 = \text{cosec}^2 x$	$\cosh^2 x - \sinh^2 x = 1$
$\sin 2x = 2\sin x \cos x$	$1 - \tanh^2 x = \text{sech}^2 x$
$\cos 2x = \cos^2 x - \sin^2 x$	$\coth^2 x - 1 = \text{cosech}^2 x$
$\cos 2x = 2\cos^2 x - 1$	$\sinh 2x = 2\sinh x \cosh x$
$\cos 2x = 1 - 2\sin^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$	$\cosh 2x = 2\cosh^2 x - 1$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	$\cosh 2x = 1 + 2\sinh^2 x$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$
$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$
$2\sin x \cos y = \sin(x + y) + \sin(x - y)$	$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$
$2\sin x \sin y = -\cos(x + y) + \cos(x - y)$	$\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$
$2\cos x \cos y = \cos(x + y) + \cos(x - y)$	

FINAL EXAMINATION

SEMESTER / SESSION: SEM II / 2014/2015
SUBJECT : CIVIL ENGINEERING MATHEMATICS I

COURSE : 1 BFF/ 2 BFF/ 3 BFF/ 4 BFF
CODE : BFC13903

Differentiation Rules	Indefinite Integrals
$\frac{d}{dx}[k]=0$, k constant	$\int k\,dx = kx + C$
$\frac{d}{dx}\left[x^n\right]=nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1}+C$, $n \neq -1$
$\frac{d}{dx}\left[\ln\|x\|\right]=\frac{1}{x}$	$\int \frac{dx}{x} = \ln\|x\| + C$
$\frac{d}{dx}[\cos x]=-\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}[\sin x]=\cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}[\tan x]=\sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}[\cot x]=-\operatorname{cosec}^2 x$	$\int \operatorname{cosec}^2 x\,dx = -\cot x + C$
$\frac{d}{dx}[\sec x]=\sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$\frac{d}{dx}[\operatorname{cosec} x]=-\operatorname{cosec} x \cot x$	$\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + C$
$\frac{d}{dx}\left[e^x\right]=e^x$	$\int e^x\,dx = e^x + C$
$\frac{d}{dx}[\cosh x]=\sinh x$	$\int \sinh x\,dx = \cosh x + C$
$\frac{d}{dx}[\sinh x]=\cosh x$	$\int \cosh x\,dx = \sinh x + C$
$\frac{d}{dx}[\tanh x]=\operatorname{sech}^2 x$	$\int \operatorname{sech}^2 x\,dx = \tanh x + C$
$\frac{d}{dx}[\coth x]=-\operatorname{cosech}^2 x$	$\int \operatorname{cosech}^2 x\,dx = -\coth x + C$
$\frac{d}{dx}[\operatorname{sech} x]=-\operatorname{sech} x \tanh x$	$\int \operatorname{sech} x \tanh x\,dx = -\operatorname{sech} x + C$
$\frac{d}{dx}[\operatorname{cosech} x]=-\operatorname{cosech} x \coth x$	$\int \operatorname{cosech} x \coth x\,dx = -\operatorname{cosech} x + C$

FINAL EXAMINATION

SEMESTER / SESSION: SEM II / 2014/2015

SUBJECT : CIVIL ENGINEERING MATHEMATICS I

COURSE : 1 BFF/ 2 BFF/ 3 BFF/ 4 BFF

CODE : BFC13903

Integration of Inverse Functions
$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C.$
$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C.$
$\int \frac{dx}{\lvert a \rvert \sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C.$
$\int \frac{dx}{\sqrt{x^2 + a^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C, \quad a > 0.$
$\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C, \quad x > a$
$\int \frac{dx}{x^2 - a^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C, & \lvert x \rvert < a \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C, & \lvert x \rvert > a \end{cases}$
$\int \frac{dx}{x\sqrt{a^2 - x^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{x}{a}\right) + C, \quad 0 < x < a.$
$\int \frac{dx}{x\sqrt{a^2 + x^2}} = -\frac{1}{a}\operatorname{cosech}^{-1}\left(\frac{x}{a}\right) + C, \quad 0 < x < a.$

Logarithm	Inverse Hiperbolic
$a^x = e^{x \ln a}$	$\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$, any x.
$\log_a x = \frac{\log_b x}{\log_b a}$	$\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right), \quad x \geq 1$
	$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right), \quad -1 < x < 1$

FINAL EXAMINATION

SEMESTER / SESSION: SEM II / 2014/2015

SUBJECT : CIVIL ENGINEERING MATHEMATICS I

COURSE : 1 BFF/ 2 BFF/ 3 BFF/ 4 BFF

CODE : BFC13903

y	$\frac{dy}{dx}$
$\sin^{-1} u$	$\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}, \quad \lvert u\rvert < 1.$
$\cos^{-1} u$	$-\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}, \quad \lvert u\rvert < 1.$
$\tan^{-1} u$	$\frac{1}{1+u^2} \cdot \frac{du}{dx}$
$\cot^{-1} u$	$-\frac{1}{1+u^2} \cdot \frac{du}{dx}$
$\sec^{-1} u$	$\frac{1}{\lvert u\rvert\sqrt{u^2-1}} \cdot \frac{du}{dx}, \quad \lvert u\rvert > 1.$
$\operatorname{cosec}^{-1} u$	$-\frac{1}{\lvert u\rvert\sqrt{u^2-1}} \cdot \frac{du}{dx}, \quad \lvert u\rvert > 1.$
$\sinh^{-1} u$	$\frac{1}{\sqrt{u^2+1}} \cdot \frac{du}{dx}$
$\cosh^{-1} u$	$\frac{1}{\sqrt{u^2-1}} \cdot \frac{du}{dx}, \quad \lvert u\rvert > 1.$
$\tanh^{-1} u$	$\frac{1}{1-u^2} \cdot \frac{du}{dx}, \quad \lvert u\rvert < 1.$
$\coth^{-1} u$	$-\frac{1}{1-u^2} \cdot \frac{du}{dx}, \quad \lvert u\rvert > 1.$
$\operatorname{sech}^{-1} u$	$-\frac{1}{u\sqrt{1-u^2}} \cdot \frac{du}{dx}, \quad 0 < u < 1.$
$\operatorname{cosech}^{-1} u$	$-\frac{1}{\lvert u\rvert\sqrt{1+u^2}} \cdot \frac{du}{dx}, \quad u \neq 0.$

FINAL EXAMINATION

SEMESTER / SESSION: SEM II / 2014/2015

COURSE : 1 BFF/ 2 BFF/ 3 BFF/ 4 BFF

SUBJECT : CIVIL ENGINEERING MATHEMATICS I

CODE : BFC13903

Area between two curves

Case 1- Integrating with respect to x: $A = \int_a^b [f(x) - g(x)]dx$

Case 2- Integrating with respect to y: $A = \int_c^d [f(y) - g(y)]dy$

Area of surface of revolution

Case 1- Revolving the portion of the curve about x-axis: $S = 2\pi \int_a^b y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

Case 2- Revolving the portion of the curve about y-axis: $S = 2\pi \int_c^d x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

Parametric curve- Revolving the curve about x-axis: $S = 2\pi \int_a^b y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Parametric curve- Revolving the curve about y-axis: $S = 2\pi \int_c^d x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Arc length

x-axis: $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

y-axis: $L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

Parametric curve: $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Curvature

Curvature, $K = \dfrac{\left|\frac{d^2y}{dx^2}\right|}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}$

Radius of curvature, $\rho = \dfrac{1}{K}$

Curvature of parametric curve

Curvature, $K = \dfrac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{(\dot{x}^2 + \dot{y}^2)^{3/2}}$

Radius of curvature, $\rho = \dfrac{1}{K}$

FINAL EXAMINATION

SEMESTER / SESSION: SEM II / 2014/2015

COURSE : 1 BFF/ 2 BFF/ 3 BFF/ 4 BFF

SUBJECT : CIVIL ENGINEERING MATHEMATICS I

CODE : BFC13903

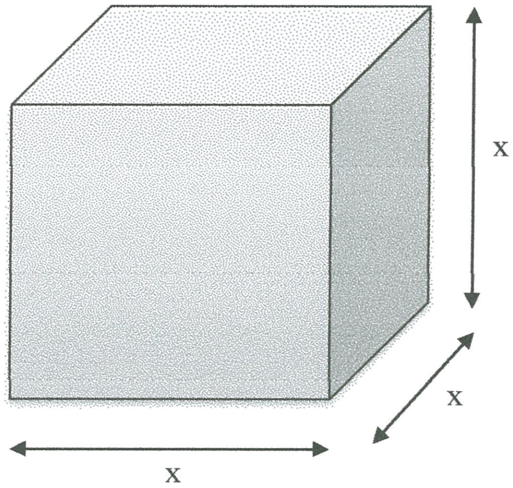

FIGURE Q2