UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2015/2016

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS IV
COURSE CODE	:	BFC 24203/ BWM 30603
PROGRAMME	:	BACHELOR OF CIVIL ENGINEERING WITH HONOURS
DATE OF EXAMINATION	:	DECEMBER 2015/JANUARY 2016
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **FOUR (4)** PAGES

Q1 The temperature distribution $u(x,t)$ of one dimensional silver rod is governed by Heat equation

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2}{\partial x^2}$$

with α^2 is thermal diffusity = 1.71.

Given the initial condition,

$$u(x,0) = \begin{cases} x, & 0 \le x \le 2, \\ 4 - x, & 2 \le x \le 4, \end{cases}$$

and boundary conditions,

$$u(0,t) = t, \quad u(4,t) = t^2.$$

Determine the temperature distribution of the rod with $\Delta x = h = 1$ and $\Delta t = k = 0.2$ for $0 \le t \le 0.4$ by using implicit Crank-Nicolson method.

(25 marks)

Q2 The steady state temperature distribution $T(x,y)$ of a thin plate over the rectangle $0 \le x \le 1, \;\; 0 \le y \le 2$, satisfies the Laplace equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0, \;\; 0 \le x \le 1, \qquad 0 \le y \le 2$$

with the boundary conditions,

$$T(0,y) = 1, \qquad T(1,y) = e^y, \qquad 0 \le y \le 2,$$
$$T(x,0) = 1, \qquad T(x,2) = e^{2x}, \qquad 0 \le x \le 1,$$

Determine the temperature distribution, $T(x,y)$ of the thin plate by using Finite-Difference method with $h = \Delta x = k = \Delta y = 0.5$.

(25 marks)

Q3 The initial-value problem $y' = \frac{2y}{x} - xy^2$, $y(1) = 5$ has a unique solution $(x) = \frac{20x^2}{5x^4-4}$.

(a) Approximate the solution at $x = 1.4$ using the fourth order Runge – Kutta method (RK4) with the same step size $h = 0.2$.

(10 marks)

(b) Briefly discuss the result of absolute error.

(6 marks)

(c) The first order initial value problem is given below

$$xy' - y = \frac{x}{x+1}$$

in interval $1 \leq x \leq 1.4$ with $h = 0.1$ and initial condition $y(1) = 0$

(i) Solve the problem by using Euler's method. (5 marks)

(ii) Find the absolute errors if the exact solution is (4 marks)

$$y(x) = x \ln\left(\frac{2x}{x+1}\right)$$

Q4 Let $y(x,t)$ denotes displacement of a vibrating string. If T is the tension in the string, ω is the weight per unit length, and g is acceleration due to gravity, then y satisfies the equation

$$\frac{\partial^2 y}{\partial t^2} = \frac{Tg}{\omega}\frac{\partial^2 y}{\partial x^2}, \qquad 0 < x < 6, \;\; t > 0$$

Suppose a particular string is 6 feet long and is fixed at both ends. Taking $T = 32$ pounds, $\omega = 0.01$ pounds/feet and $g = 32$ feet/sec^2. The initial conditions are

$$y(x,0) = \begin{cases} \frac{x}{6}, & 0 \leq x \leq 3 \\ \frac{6-x}{6}, & 3 \leq x \leq 6 \end{cases} \qquad and \quad \frac{\partial y}{\partial t}(x,0) = x(x-6)$$

(a) Arrange all boundary conditions into grid.

(4 marks)

(b) Solve for y up to level 1 only by using the Finite-Difference method
Performed all calculation with $\Delta x = 1$ feet and $\Delta t = 0.01$ seconds.

(21 marks)

– **END OF QUESTION** –

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FORMULAS

Fourth-order Runge-Kutta method: $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$ $\quad k_2 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right)$

$k_3 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right)$ $\quad k_4 = hf(x_i + h, y_i + k_3)$

Finite difference method:

$$y_i' \approx \frac{y_{i+1} - y_{i-1}}{2h}, \quad y_i'' \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

Heat equation – Implicit Crank-Nicolson:

$$\left(\frac{\partial u}{\partial t}\right)_{i,j+\frac{1}{2}} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j+\frac{1}{2}}$$

$$\frac{u_{i,j+\frac{1}{2}} - u_{i,j}}{k} = \frac{c^2}{2}\left(\frac{u_{i+j,j+1} - 2u_{i,j+1} + u_{i-1,j+1}}{h^2} + \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2}\right)$$

Laplace Equation: Finite-Difference method:

Laplace's equation at the point (x_i, y_i)

$$\frac{\partial^2}{\partial x^2} u(x_i, y_i) + \frac{\partial^2}{\partial y^2} u(x_i, y_i) = 0$$

$$\left(\frac{\partial^2 u}{\partial u^2}\right)_{i,j} + \left(\frac{\partial^2 u}{\partial y^2}\right)_{i,j} = 0$$

Central space (CTCS) finite-difference approximation:

$$\frac{u_{i+1,j} - 2u_{1,j} + u_{i-1,j}}{k^2} = c^2 \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{h^2}$$

Euler's Method: $y(x_{i+1}) = y(x_i) + hy'(x_i)$

$$y_{i+1} = y_i + hf(x_i, y_i)$$