UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2015/2016

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS III
COURSE CODE	:	BFC 24103 / BWM 20403
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	JUNE/JULY 2016
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ONE (1)** QUESTION IN **PART A** AND **ALL** QUESTIONS IN **PART B**

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

PART A

Q1 (a) Determine f_x, f_y and f_{xx} of the function

$$f(x,y) = e^{xy}\sin(4y^2)$$

(5 marks)

(b) Given $f(x,y) = \sqrt{4 + x^2 + 2y^2}$. Use the total differential to approximate the value of $f(-0.07, 2.98)$ by taking $(0, 3)$ as a guide point.

(7 marks)

(c) (i) Show that the limit of the function $f(x,y) = \frac{xy^2}{x^2+y^4}$ does not exist when $(x,y) \to (0,0)$ by taking the limit along a straight line $y = mx$ and the parabola $x = y^2$.

(7 marks)

(ii) Determine whether the function

$$f(x,y) = \begin{cases} \dfrac{x^4 - 4y^4}{x^2 + 2y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0) \end{cases}$$

continuous at (0,0) or not.

(6 marks)

Q2 (a) By using polar coordinates, evaluate the following integration

$$\iint_R (x^2 + y^2 - 2x)\,dA$$

where R is the region bounded by the x-axis, the line $y = \frac{1}{2}x$ and the circle $x^2 + y^2 = \frac{1}{4}$

(10 marks)

(b) A hole in the shape of a cone $z = \sqrt{x^2 + y^2}$ is drilled out from a sphere $x^2 + y^2 + z^2 = 25$. By using spherical coordinates, determine the volume of the remaining solid.

(7 marks)

(c) A solid of half circular cylinder $y = -\sqrt{4 - x^2}$ is bounded by $z = 2$ and $z = 10 - y$. Let assume that the solid has density $\delta(x,y,z) = x^2 + y^2$, determine the mass of the solid.

(8 marks)

PART B

Q3 (a) Given the vector-valued function

$$\mathbf{r}(t) = \sqrt{2-t}\,\mathbf{i} + \frac{e^t - 1}{t}\mathbf{j} + \ln(t+1)\,\mathbf{k}$$

(i) Find the domain of $\mathbf{r}(t)$

(7 marks)

(ii) Determine $\lim\limits_{t\to 0} \mathbf{r}(t)$

(7 marks)

(b) Find a vector equation that represents the curve of intersection of the cylinder $x^2 + y^2 = 16$ and the plane $x + z = 5$

(4 marks)

(c) Determine the arc length of the helix $\mathbf{r}(t) = b\cos t\,\mathbf{i} + b\sin t\,\mathbf{j} + t\sqrt{1-b^2}\,\mathbf{k}$, from $t = 0$ to 2π, where b is constant

(7 marks)

Q4 (a) Use Green's Theorem to evaluate the line integral $\oint_C (x + y^2)\,dx + 2x\,dy$

where C is the boundary of the region between $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$

(15 marks)

(b) Determine the work done by force field $\mathbf{F}(x,y) = x\mathbf{i} + (2x+y)\mathbf{j}$ along the curve C, where C is the upper semicircle that starts from $(1, 0)$ and ends at $(0, 1)$

(10 marks)

Q5 (a) Given the vector field $\mathbf{F}(x,y,z) = x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}$ across the surface S. S is enclosed by tetrahedron in the first octant bounded by $x + 2y + z = 1$ and the coordinate planes. Use Gauss's Theorem to calculate the outward flux

$$\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$$

(15 marks)

(b) Given the vector field $\mathbf{F}(x,y,z) = 2x\,\mathbf{i} + 3x^2\,\mathbf{j} + 3z^2\,\mathbf{k}$ across the surface S which is part of a sphere $x^2 + y^2 + z^2 = 4$ for which $z \geq 0$ with upward orientation. Use Stokes' Theorem to evaluate

$$\int_C \mathbf{F}\,d\mathbf{r}$$

where C to be the positively oriented circle $x^2 + y^2 = 4$ that forms the boundary of S in the xy-plane

(10 marks)

– END OF QUESTION –

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FORMULAE:

Relative extrema test:

Let Discriminant = $g(x, y) = f_{xx}(x, y) - [f_{xy}(x, y)]^2$ and (a, b) is critical point

(i) If $g(a, b) > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is local minimum

(ii) If $g(a, b) > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is local maximum

(iii) If $g(a, b) < 0$, then $(a, b, f(a, b))$ is saddle point

(iv) If $g(a, b) = 0$, then the test is inconclusive

Cartesian coordinates to Polar coordinates:

$x = r\cos\theta, y = r\sin\theta, x^2 + y^2 = r^2, \tan\theta = \dfrac{y}{x}$ and $0 \le \theta \le 2\pi$

$$\iint_R f(x,y)\,dA = \int_{\theta=\theta_1}^{\theta=\theta_2}\int_{r=g_1(\theta)}^{r=g_2(\theta)} f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$$

Cartesian coordinates to Cylindrical coordinates:

$x = r\cos\theta, y = r\sin\theta, z = z, x^2 + y^2 = r^2, \tan\theta = \dfrac{y}{x}$ and $0 \le \theta \le 2\pi$

$$\iiint_G f(x,y,z)\,dV = \int_{\theta=\theta_1}^{\theta=\theta_2}\int_{r=h_1(\theta)}^{r=h_2(\theta)}\int_{z=g_1(r,\theta)}^{z=g_2(r,\theta)} f(r\cos\theta, r\sin\theta, z)\,r\,dz\,dr\,d\theta$$

Cartesian coordinates to Spherical coordinates:

$x = \rho\sin\emptyset\cos\theta, y = \rho\sin\emptyset\sin\theta, z = \rho\cos\emptyset, x^2 + y^2 + z^2 = \rho^2, 0 \le \emptyset \le \pi$ and $0 \le \theta \le 2\pi$

$$\iiint_G f(x,y,z)\,dV = \int_{\theta=\theta_1}^{\theta=\theta_2}\int_{\emptyset=h_1(\theta)}^{\emptyset=h_2(\theta)}\int_{\rho=g_1(r,\theta)}^{\rho=g_2(r,\theta)} f(\rho\sin\emptyset\cos\theta, \rho\sin\emptyset\sin\theta, \rho\cos\emptyset)\rho^2\sin\emptyset\,d\rho\,d\emptyset\,d\theta$$

Partial derivatives of f with respect to x:

$$f_x, f_x(x,y) = \lim_{h\to 0}\frac{f(x+h,y) - f(x,y)}{h}$$

Partial derivatives of f with respect to y:

$$f_y, f_y(x,y) = \lim_{h\to 0}\frac{f(x,y+h) - f(x,y)}{h}$$

The second-order partial derivatives for $f(x, y)$:

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x\partial x} = f_{xx}$$

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The total differential of z:

$$d_z = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

Triple integrals in 3-dimensional Cartesian Coordinate (x, y, z):

$$V = \iiint_G dV = \iiint_G dz\,dy\,dx$$

Triple integrals in Cylindrical Coordinate (r, θ, z):

$$V = \iiint_G dV = \iiint_G dz\,r\,dr\,d\theta$$

$x = r\cos\theta$, $y = r\sin\theta$, z = z and $x^2 + y^2 = r^2$

Centroid for a homogeneous lamina:

$$\bar{x} = \frac{1}{area}\iint_R x\,dA, \quad \bar{y} = \frac{1}{area}\iint_R y\,dA$$

Unit Tangent Vector, $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

Principal Unit Normal Vector, $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

Arc length of C in the interval [a, b], $s = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\; dt = \int_a^b \|\mathbf{r}'(t)\|\,dt$

Curvature of C, $\kappa = \dfrac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|}$

Green Theorem: $\oint_C M\,dx + N\,dy = \iint_R \left(\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}\right) dA$

Gauss Theorem: $\iint_S \mathbf{F} \bullet \mathbf{n}\, dS = \iiint_G \nabla \bullet \mathbf{F}\, dV$

Stokes Theorem: $\oint_C \mathbf{F} \bullet d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \bullet \mathbf{n}\, dS$

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Surface Integral:

Let S be a surface with equation $z = g(x, y)$ and let R be its projection on the xy-plane.

$$\iint_S f(x,y,z)\,dS = \iint_R f(x,y,g(x,y))\sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\,dA$$

$$\iint_S \mathbf{F}\cdot\mathbf{n}\,dS = \iint_R \mathbf{F}\cdot\left[-\frac{\partial z}{\partial x}\mathbf{i}-\frac{\partial z}{\partial y}\mathbf{j}+\mathbf{k}\right]dA, \text{ oriented upward}$$

$$\iint_S \mathbf{F}\cdot\mathbf{n}\,dS = \iint_R \mathbf{F}\cdot\left[+\frac{\partial z}{\partial x}\mathbf{i}+\frac{\partial z}{\partial y}\mathbf{j}-\mathbf{k}\right]dA, \text{ oriented downward}$$