Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2015/2016

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS II
COURSE CODE	:	BFC 14003 / BWM 10203
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	JUNE / JULY 2016
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B**

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

PART A

Q1 (a) Apply the alternating series test to show that the following series converge:

$$\sum_{n=1}^{\infty} \frac{2^n}{n^5\sqrt{n}}$$

(5 marks)

(b) Determine whether the series is absolutely convergent, conditionally convergent or divergent:

$$\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n\sqrt{n}}$$

(5 marks)

(c) Indicate the **n**$^{\text{th}}$ Maclaurin polynomials for:

$$f(x) = \frac{1}{1-2x}$$

(5 marks)

(d) By using integration of power series, calculate:

$$f(x) = -\frac{1}{2}\ln(1+2x)$$

(5 marks)

Q2 (a) Calculate the Fourier series for $f(x) = L - x$ on $-L \leq x \leq L$.

(10 marks)

(b) Point out the Fourier series for $f(x) = x^2$ on $-L \leq x \leq L$ and classify the given function whether in odd or even function.

(10 marks)

PART B

Q3 (a) Identify whether or not the following equations is homogeneous:

$$\frac{dy}{dx} = \frac{3xy + y^2}{3x^2}$$

If homogeneous, find the solution of the equation.

(6 marks)

(b) Calculate the solution of the differential equation $\frac{dy}{dx} = \frac{y-1}{x+3}$ which satisfies an initial condition $y(-1) = 0$ by using separable equation.

(7 marks)

(c) Convert the following differential equation to a linear equation:

$$\frac{1}{x}\frac{dy}{dx} - \frac{2y}{x^2} = x\cos x \;, x > 0$$

Hence, find the solution of the original equation which satisfies the initial condition $y = 1 \; and \; x = 2$.

(7 marks)

Q4 (a) The population rate of a bacteria culture grows is proportional to the number of bacteria present. If the number of bacteria grew from 500 to 2500 in 12 hours, estimate the number of bacteria after 24 hours.

(10 marks)

(b) The temperature inside a refrigerator is maintained at 4°C. An object at 80°C is placed in the refrigerator to cool. After 110 seconds, its temperature drops to 45°C. Calculate the time will be taken for the temperature to drop to 15°C by using Newton's Law of Cooling.

(10 marks)

Q5 (a) Interpret $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 6e^x$ by using the method of undetermined coefficient.

(8 marks)

(b) Given $y'' - 2y' + 2y = e^x(1 + \sin x)$. Produce the solution for the given differential equation by using the variation of parameters method.

(12 marks)

Q6 **<u>FIGURE Q6</u>** shows a block with mass of 0.65 kg attached to a spring, and the spring stretches 0.43 m to bring the system equilibrium. The block is then pulled down an additional 0.15 m from its equilibrium position and released with a downward velocity of 0.08 m/s.

(a) Identify the position of the block, for $t \geq 0$.

(12 marks)

(b) If the support of the spring vibrate and produce an external force of $F_{ext} = \cos \omega t$, sketch the graph for $\omega = 4$ and $t \geq 0$. Assuming $\omega \neq \omega_o$ and $y(0) = y'(0) = 0$.

(8 marks)

- END OF QUESTIONS –

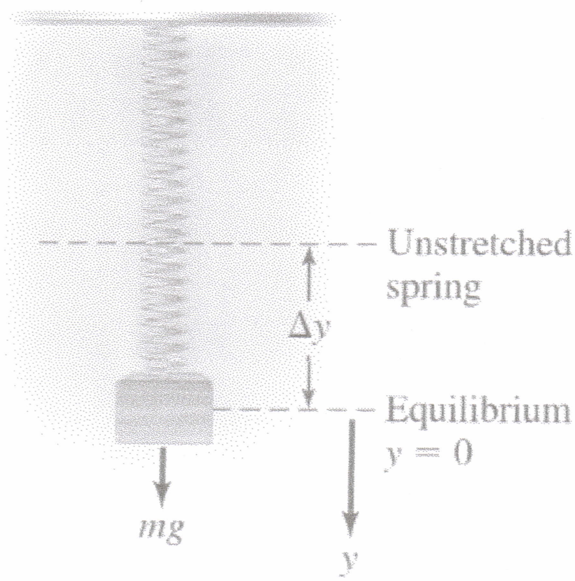

FIGURE Q6

FINAL EXAMINATION

SEMESTER / SESSION: SEM II / 2015/2016 PROGRAMME CODE: BFF
COURSE NAME: CIVIL ENGINEERING MATHEMATICS II COURSE CODE: BFC 14003 / BWM 10203

Formulae

Characteristic Equation and General Solution

Case	Roots of the Characteristic Equation	General Solution
1	m_1 and m_2 ; real and distinct	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	$m_1 = m_2 = m$; real and equal	$y = (A + Bx)e^{mx}$
3	$m = \alpha \pm i\beta$; imaginary	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Undetermined Coefficients

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + \cdots + A_1 x + A_0$	$x^r(B_n x^n + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Variation of Parameters

Wronskian	Parameter	Solution
$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$	$u_1 = -\int \frac{y_2 f(x)}{W} dx, \quad u_2 = \int \frac{y_1 f(x)}{W} dx$	$y_p = u_1 y_1 + u_2 y_2$

Fourier Series

Fourier series expansion of periodic function with period $2L$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$

where

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$

Fourier half-range series expansion

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$

where

$$a_0 = \frac{2}{L}\int_{0}^{L} f(x)\,dx$$

$$a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$b_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$

FINAL EXAMINATION

SEMESTER / SESSION: SEM II / 2015/2016
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Laplace Transforms

$\mathcal{L}\{f(t)\}=\int_0^\infty f(t)e^{-st}dt=F(s)$			
$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
$t^n,\ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
e^{at}	$\frac{1}{s-a}$	$\delta(t-a)$	e^{-as}
$\sin at$	$\frac{a}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\int_0^t f(u)g(t-u)du$	$F(s).G(s)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$y(t)$	$Y(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$\dot{y}(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$\ddot{y}(t)$	$s^2Y(s)-sy(0)-\dot{y}(0)$
$t^n f(t),\ n=1,2,3,\ldots$	$(-1)^n\frac{d^nF(s)}{ds^n}$		

Periodic Function for Laplace transform : $\mathcal{L}\{f(t)\}=\frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)dt,\quad s>0.$

FINAL EXAMINATION

SEMESTER / SESSION: SEM II / 2015/2016
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Trigonometric and Hyperbolic Identities

Trigonometric	Hiperbolic
$\cos^2 x+\sin^2 x=1$	$\sinh x=\dfrac{e^x-e^{-x}}{2}$
$1+\tan^2 x=\sec^2 x$	$\cosh x=\dfrac{e^x+e^{-x}}{2}$
$\cot^2 x+1=\operatorname{cosec}^2 x$	$\cosh^2 x-\sinh^2 x=1$
$\sin 2x=2\sin x\cos x$	$1-\tanh^2 x=\operatorname{sech}^2 x$
$\cos 2x=\cos^2 x-\sin^2 x$	$\coth^2 x-1=\operatorname{cosech}^2 x$
$\cos 2x=2\cos^2 x-1$	$\sinh 2x=2\sinh x\cosh x$
$\cos 2x=1-2\sin^2 x$	$\cosh 2x=\cosh^2 x+\sinh^2 x$
$\tan 2x=\dfrac{2\tan x}{1-\tan^2 x}$	$\cosh 2x=2\cosh^2 x-1$
$\sin(x\pm y)=\sin x\cos y\pm\cos x\sin y$	$\cosh 2x=1+2\sinh^2 x$
$\cos(x\pm y)=\cos x\cos y\mp\sin x\sin y$	$\tanh 2x=\dfrac{2\tanh x}{1+\tanh^2 x}$
$\tan(x\pm y)=\dfrac{\tan x\pm\tan y}{1\mp\tan x\tan y}$	$\sinh(x\pm y)=\sinh x\cosh y\pm\cosh x\sinh y$
$2\sin x\cos y=\sin(x+y)+\sin(x-y)$	$\cosh(x\pm y)=\cosh x\cosh y\pm\sinh x\sinh y$
$2\sin x\sin y=-\cos(x+y)+\cos(x-y)$	$\tanh(x\pm y)=\dfrac{\tanh x\pm\tanh y}{1\pm\tanh x\tanh y}$
$2\cos x\cos y=\cos(x+y)+\cos(x-y)$	

FINAL EXAMINATION

SEMESTER / SESSION: SEM II / 2015/2016 PROGRAMME CODE: BFF
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Differentiation and Integration

Differentiation Rules	Indefinite Integrals
$\frac{d}{dx}[k]=0$, k constant	$\int k\,dx = kx + C$
$\frac{d}{dx}\left[x^n\right]=nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1}+C$, $n \neq -1$
$\frac{d}{dx}\left[\ln\lvert x\rvert\right]=\frac{1}{x}$	$\int \frac{dx}{x} = \ln\lvert x\rvert + C$
$\frac{d}{dx}[\cos x]=-\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}[\sin x]=\cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}[\tan x]=\sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}[\cot x]=-\text{cosec}^2 x$	$\int \text{cosec}^2 x\,dx = -\cot x + C$
$\frac{d}{dx}[\sec x]=\sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$\frac{d}{dx}[\text{cosec}\, x]=-\text{cosec}\, x \cot x$	$\int \text{cosec}\, x \cot x\,dx = -\text{cosec}\, x + C$
$\frac{d}{dx}\left[e^x\right]=e^x$	$\int e^x\,dx = e^x + C$
$\frac{d}{dx}[\cosh x]=\sinh x$	$\int \sinh x\,dx = \cosh x + C$
$\frac{d}{dx}[\sinh x]=\cosh x$	$\int \cosh x\,dx = \sinh x + C$
$\frac{d}{dx}[\tanh x]=\text{sech}^2 x$	$\int \text{sech}^2 x\,dx = \tanh x + C$
$\frac{d}{dx}[\coth x]=-\text{cosech}^2 x$	$\int \text{cosech}^2 x\,dx = -\coth x + C$
$\frac{d}{dx}[\text{sech}\, x]=-\text{sech}\, x \tanh x$	$\int \text{sech}\, x \tanh x\,dx = -\text{sech}\, x + C$
$\frac{d}{dx}[\text{cosech}\, x]=-\text{cosech}\, x \coth x$	$\int \text{cosech}\, x \coth x\,dx = -\text{cosech}\, x + C$