Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2013/2014

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS IV
COURSE CODE	:	BFC 24203
PROGRAMME	:	2 BFF/ 3 BFF
EXAMINATION DATE	:	JUNE 2014
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS IN **SECTION A** AND **TWO (2)** QUESTIONS IN **SECTION B.** B) ALL CALCULATIONS AND ANSWERS MUST BE IN **THREE (3) DECIMAL PLACES**.

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

SECTION A

Q1 (a) Given the heat equation

$$\frac{\partial u}{\partial t} = 8\frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < 1, \qquad t > 0,$$

with the boundary conditions, $u(0,t) = 0$ and $u(1,t) = 2$
and the initial condition $u(x,0) = x(1+x)$.
By using explicit method, solve the heat equation up to first level $(t \leq 0.01)$ and by taking $\Delta x = h = 0.2$ and $\Delta t = k = 0.01$.

(12 marks)

(b) Form the system of linear equation, $Ax = b$ for the Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

with boundary conditions,

$$u(x,y) = \begin{cases} 1 & \text{if } x = 0, \text{ and } 0 < y < 1, \\ 1 & \text{if } x = 1, \text{ and } 0 < y < 1, \\ 0 & \text{if } y = 0, \text{ and } 0 < x < 1, \\ 1 & \text{if } y = 1, \text{ and } 0 < x < 1. \end{cases}$$

by taking $\Delta x = h = 0.25$ and $\Delta y = k = 0.5$.
(**DO NOT SOLVE** the system of linear equations)

(13 marks)

Q2 Consider a fin of length 5 unit has three nodes and two elements, as shown in **FIGURE Q2**. The heat flow equation is:

$$\frac{d}{dx}\left(A(x)k(x)\frac{dT(x)}{dx}\right) + Q(x) = 0, \quad \text{for} \quad 0 \leq x \leq 5$$

with $A(x)$ is the cross-sectional area, $k(x)$ is the thermal conductivity, $T(x)$ is the temperature at length x and $Q(x)$ is the heat supply per unit time and per unit length. Find the temperature at each nodal point, T_2 and T_3, if $A(x)$ is 30 unit, $k(x)$ is 10 unit and $Q(x)$ is 10 unit. Let the temperature at $x = 0$ is 0 unit and the heat flux, $-k\left.\frac{dT}{dx}\right|_{x=5} = 10$ unit.

(25 marks)

SECTION B

Q3 (a) A small village always used a water tank with radius $r = 10$ m to supply clean water to the village. The volume of water in it can be computed by the formula

$$V = \pi h^2 \frac{(3r - h)}{3}$$

where V, h and r are volume of water, depth of water and radius of tank, respectively. What is the depth of water in the tank so that the volume of water in it is 1000 m^3? Use Newton-Raphson method to solve this problem with the initial guess of 6 m.

(10 marks)

(b) Solve the system of linear equations below by using Gauss-Seidel iteration method

$$\begin{aligned} 2x_1 + 10x_2 + 3x_3 &= 44 \\ 3x_1 + 4x_2 + 11x_3 &= 59 \\ 9x_1 + 2x_2 + 3x_3 &= 16 \end{aligned}$$

by taking initial guess as $x^{(0)} = [0 \quad 0 \quad 0]^T$,

and iterate until $\max\left\{\left|x_i^{(k+1)} - x_i^{(k)}\right|\right\} < \varepsilon = 0.005$.

(15 marks)

Q4 (a) Relationship between the applied force P and displacement u of the three-bar truss structure in **FIGURE Q4 (a)** is nonlinear according to the data given in table below. Derive the Lagrange polynomial for the given data points. Then estimate the force at displacements of 0.17.

P (N)	u (cm)
0.00	0.00
9.80	0.25
12.00	0.50
14.20	0.75

(10 marks)

(b) Given a set of data is in the following table. By using step size, $h = 0.1$ find approximate value for the first derivative of $f(x)$ at $x = 3.3$ using **ALL APPROPRIATE** difference formula.

x	3.0	3.1	3.2	3.3	3.4
$f(x)$	0.588	0.736	0.911	1.382	1.795

(8 marks)

(c) Use suitable Simpson's rule to approximate $\int_0^5 \sqrt{x^3 + 5}dx$ using $n = 10$.

(7 marks)

Q5 (a) Given that $A = \begin{pmatrix} 7 & 1 & 0 \\ 1 & 6 & 2 \\ 0 & 2 & 9 \end{pmatrix}$ with three positive eigenvalues. By using the initial vector $\mathbf{v}^{(0)} = (0 \quad 1 \quad 1)^T$ and convergence criterion $|m_{k+1} - m_k| < \varepsilon = 0.005$.

(i) Compute the dominant eigenvalue and the corresponding eigenvector of the matrix A using Power method.

(ii) Use the Shifted Power method to compute the smallest eigenvalue and the corresponding eigenvector of the matrix A.

(16 marks)

(b) Given the initial value problem

$$\frac{dy}{dx} + 4xy = 6x, \; y(0) = 1$$

Approximate the value $y(0.2)$ using

(i) Euler's method with $h = 0.1$, and

(ii) Runge-Kutta method of order 4 with $h = 0.2$.

(9 marks)

- END OF QUESTION -

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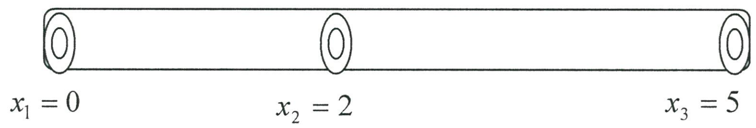

FIGURE Q2

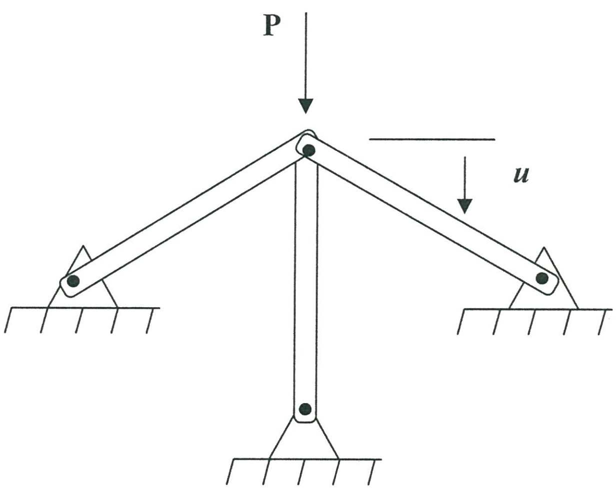

FIGURE Q4(a)

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FORMULAS

Nonlinear equations

Newton-Raphson method : $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}$, $i = 0,1,2,\ldots$

System of linear equations

Gauss-Seidel iteration : $x_i^{(k+1)} = \dfrac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}}{a_{ii}}$, $\forall i = 1, 2, 3,.., n$.

Interpolation

Lagrange interpolation : $P_n(x) = \sum_{i=0}^{n} L_i(x) f_i$ for $k = 0,1,2,3,\ldots,n$ with

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{(x - x_j)}{(x_i - x_j)}$$

Numerical Differentiation

2-point forward difference: $f'(x) \approx \dfrac{f(x+h) - f(x)}{h}$

2-point backward difference: $f'(x) \approx \dfrac{f(x) - f(x-h)}{h}$

3-point central difference: $f'(x) \approx \dfrac{f(x+h) - f(x-h)}{2h}$

3-point forward difference: $f'(x) \approx \dfrac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h}$

3-point backward difference: $f'(x) \approx \dfrac{3f(x) - 4f(x-h) + f(x-2h)}{2h}$

5-point difference formula: $f'(x) \approx \dfrac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}$

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Numerical Integration

Simpson $\frac{1}{3}$ Rule: $\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i\text{ odd}}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i\text{ even}}}^{n-2} f_i \right]$

Simpson $\frac{3}{8}$ rule : $\int_a^b f(x)\,dx \approx \frac{3}{8}h\left[\begin{array}{c} (f_0 + f_n) + 3(f_1 + f_2 + f_4 + f_5 + \cdots + f_{n-2} + f_{n-1}) \\ + 2(f_3 + f_6 + \cdots + f_{n-3}) \end{array} \right]$

Eigen value

Power Method: $v^{(k+1)} = \frac{1}{m_{k+1}} A v^{(k)}, \qquad k = 0, 1, 2, \ldots..$

Ordinary Differential Equation

Initial Value Problem:

Euler's Method: $y_{i+1} = y(x_i) + hf(x_i, y_i)$

Fourth-order Runge-Kutta Method: $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$ $\qquad k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$ $\qquad k_4 = hf(x_i + h, y_i + k_3)$

Partial Differential Equation
Heat Equation: Finite Difference Method

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \qquad \frac{u_{i,j+1} - u_{i,j}}{k} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

Laplace Equation: Finite Difference Method

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j} + \left(\frac{\partial^2 u}{\partial y^2}\right)_{i,j} = 0 \qquad \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = 0$$

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Finite Element Method

Heat flow problem in 1 dimension for $a \le x \le b$

$N(x) = [N_1(x) \;\; N_2(x) \;\; \cdots \;\; N_n(x)]$

$N_m(x) = [N_m^e(x) \;\;]$ is global shaped function for element e at node m

$$T = \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_n \end{bmatrix}$$ is the temperature vector at node

$$\mathbf{K}T = \mathbf{f}_\mathrm{b} - \mathbf{f}_\mathrm{L}$$

where

stiffness matrix, $\mathbf{K} = \int_a^b \mathbf{B}^T Ak\mathbf{B}\, dx$ **or**

$$K_{ij} = \int_a^b \frac{dN_i}{dx} Ak \frac{dN_j}{dx}\, dx$$ is a square matrix with dimension $n \times n$,

boundary vector, $\mathbf{f}_\mathrm{b} = \left[\mathbf{N}_i A(x) k(x) \frac{dT}{dx} \right]_a^b$ have the dimension $n \times 1$,

load vector, $\mathbf{f}_\mathrm{L} = -\int_a^b \mathbf{N}_i Q(x)\, dx$ have the dimension $n \times 1$.