UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2013/2014

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS II
COURSE CODE	:	BFC 14003
PROGRAMME	:	1 BFF / 2 BFF
EXAMINATION DATE	:	JUNE 2014
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER ALL QUESTIONS IN **SECTION A** B) ANSWER **FOUR** (4) QUESTIONS IN **SECTION B**

THIS QUESTION PAPER CONSISTS OF **TEN (10)** PAGES

SECTION A

Q1 A periodic function $f(x)$ is defined by

$$f(x)=\begin{cases}-4, & -\pi \le x < 0,\\ 4, & 0 \le x < \pi,\end{cases}$$

and

$$f(x) = f(x+2\pi),$$

which it can be written as a Fourier series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx).$$

(a) Sketch the graph of $f(x)$ over $-3\pi \le x \le 3\pi$.

(3 marks)

(b) Calculate the Fourier coefficients, a_0 , a_n and b_n.

(10 marks)

(c) Show that the Fourier series expansion for $f(x)$ is

$$f(x) = \frac{16}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2n-1)x}{2n-1}.$$

(3 marks)

(d) By referring to the periodic function $f(x)$ above and use $x = \frac{\pi}{2}$, deduce the sum of the infinite series

$$1 - \frac{1}{3} + \frac{1}{5} - \cdots.$$

(4 marks)

SECTION B

Q2 (a) Form the differential equation of the family of curves $y = ae^{3x} + be^{x}$ where a and b are constants.

(5marks)

(b) Solve the following differential equation by using appropriate method.

(i) $x\dfrac{dy}{dx} - y = x^3 + 3x^2 - 2x$.

(3 marks)

(ii) $\dfrac{dy}{dx} = \dfrac{xy}{x^2+1}$, $y(0) = 2$.

(4 marks)

(c) A thermometer at a temperature of $72^0 F$ is taken from a room and placed outside where the room temperature is $32^0 F$. If after half minutes the thermometer temperature is $50^0 F$.

(i) What is the temperature of the thermometer after a minute?

(6 marks)

(ii) How long it will take for the thermometer to reach a temperature of $35^0 F$?

(2 marks)

Q3 (a) Find the general solution for the second order differential equation

$$y'' + 6y' + 9y = \sin 2t.$$

by using the undetermined coefficient method.

(7 marks)

(b) Given a non-homogeneous second order differential equation

$$\ddot{y} + 2\dot{y} + 5y = xe^{-x}.$$

Find the general solution for the equation by using variation of parameters method.

(5 marks)

(c) A spring is stretched 0.49 m ($\Delta\ell$) when a 6 *kg* mass (m) is attached. The weight is then pulled down an additional 0.8 m and released with an upward velocity of 10 ms^{-1}. Neglect the damping constant, *c*. If the general equation describing the spring-mass system is

$$m\frac{d^2y}{dt^2}+c\frac{dy}{dt}+ky=0,$$

find an equation for the position of the spring at any time *t*.

(Hints : Weight, $W=mg$, $k=\dfrac{W}{\Delta\ell}$, $g\approx 9.8\,\text{ms}^{-2}$)

(8 marks)

Q4 (a) Solve the following.

(i) $\mathcal{L}\{5e^{2t}-4t^2+\cosh 3t\}$.

(3 marks)

(ii) $\mathcal{L}\{e^{-2t}\sin 5t\}$.

(3 marks)

(iii) $\mathcal{L}^{-1}\left\{\dfrac{s-(\frac{3+\sqrt{7}}{2})}{s^2-3s+4}+\dfrac{2}{(s-4)^5}\right\}$.

(5 marks)

(b) An object with mass *m* is placed upon the lower end of a spring suspended from the ceiling. The object comes to rest in its equilibrium position. Beginning at $t=0$, an external forces given by $F(t)=\sin 2t$ is applied to the system. Given that the damping forces is $2\dfrac{dy}{dt}$, $m=1$ (lb) and the spring constant, *k* is 3 lb/ft.

(i) State the initial conditions when $y(0)$ and $y'(0)$.

(2 marks)

(ii) The general equation describes the system as

$$m\frac{d^2y}{dt^2}+b\frac{dy}{dt}+ky=F(t)$$

where m is mass, $b\dfrac{dy}{dt}$ is damping forces, k is spring constant and $F(t)$ is external forces.

Show that,

$$Y(s) = -\frac{2}{17}\left[\frac{1+2s}{s^2+2^2}\right] + \frac{1}{17}\left[\frac{6+4(s+1)}{(s+1)^2+2}\right]$$

(5 marks)

(iii) Hence, determine the resulting motion, $y(t)$.

(2 marks)

Q5 (a) Find the third degree Taylor polynomials of $f(x)=\sqrt{x}$, at a point $x_0 = 3$.

(4 marks)

(b) Determine whether the series $\displaystyle\sum_{n=2}^{\infty}\frac{\sin n}{n^2}$ converges or diverges.

(3 marks)

(c) Find the interval of convergence and the radius of convergence for given series

$$\sum_{n=1}^{\infty}\frac{n^3(x+5)^n}{6^n}.$$

(6 marks)

(d) Given that the power series of $f(x)=xe^x$ is $\displaystyle\sum_{k=0}^{\infty}\frac{x^{k+1}}{k!}$.

(i) Evaluate $\displaystyle\int_0^1 xe^x\,dx$.

(2 marks)

(ii) By using the answer in **Q5(d)(i)** show that $\displaystyle\sum_{k=1}^{\infty}\frac{1}{k!(k+2)} = \frac{1}{2}$.

(5 marks)

Q6 (a) A periodic function $f(x)$ is defined by

$$f(x)=\begin{cases}0, & -\pi<x<0,\\ \pi-x, & 0<x<\pi,\end{cases}$$

and

$$f(x)=f(x+2\pi).$$

(i) Sketch the graph of $f(x)$ over $-3\pi<x<3\pi$.

(3 marks)

(ii) State whether the function is even, odd or neither.

(1 marks)

(b) Determine whether the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt[4]{n}}$ converges absolutely, converges conditionally, or diverges by using a suitable convergences test.

(6 marks)

(c) Given that, $\frac{1}{1-x}=\sum_{n=0}^{\infty}x^n$. Find the power series of $\frac{1}{(1+x)^2}$.

(6 marks)

(d) Obtain the Maclaurin series expansion of $f(x)=\sin x$. Write your answer in summation form.

(4 marks)

- END OF QUESTION -

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Formula

Second-order Differential Equation

The roots of characteristic equation and the general solution for differential equation $ay'' + by' + cy = 0$.

Characteristic equation: $am^2 + bm + c = 0$.		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y = Ae^{m_1 x} + Be^{m_2 x}$
2.	Real and equal roots: $m = m_1 = m_2$	$y = (A + Bx)e^{mx}$
3.	Complex roots: $m_1 = \alpha + \beta i$, $m_2 = \alpha - \beta i$	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

The method of undetermined coefficients

For non-homogeneous second order differential equation $ay'' + by' + cy = f(x)$, the particular solution is given by $y_p(x)$:

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + A_{n-1}x^{n-1} + \cdots + A_1 x + A_0$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(P\cos\beta x + Q\sin\beta x)$
$P_n(x)e^{\alpha x}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)e^{\alpha x}$
$P_n(x)\begin{cases}\cos\beta x \\ \sin\beta x\end{cases}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)\cos\beta x +$ $x^r(C_n x^n + C_{n-1}x^{n-1} + \cdots + C_1 x + C_0)\sin\beta x$
$Ce^{\alpha x}\begin{cases}\cos\beta x \\ \sin\beta x\end{cases}$	$x^r e^{\alpha x}(P\cos\beta x + Q\sin\beta x)$
$P_n(x)e^{\alpha x}\begin{cases}\cos\beta x \\ \sin\beta x\end{cases}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)e^{\alpha x}\cos\beta x +$ $x^r(C_n x^n + C_{n-1}x^{n-1} + \cdots + C_1 x + C_0)e^{\alpha x}\sin\beta x$

Note : r is the least non-negative integer (r = 0, 1, or 2) which determine such that there is no terms in particular integral $y_p(x)$ corresponds to the complementary function $y_c(x)$.

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The method of variation of parameters

If the solution of the homogeneous equation $ay'' + by' + cy = 0$ is $y_c = Ay_1 + By_2$, then the particular solution for $ay'' + by' + cy = f(x)$ is

$$y = uy_1 + vy_2,$$

where $u = -\int \frac{y_2 f(x)}{aW} dx + A,\ v = \int \frac{y_1 f(x)}{aW} dx + B$ and $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$.

Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
e^{at}	$\frac{1}{s-a}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\sin at$	$\frac{a}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\cos at$	$\frac{s}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$y(t)$	$Y(s)$
$t^n,\ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$y'(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$
$t^n f(t),\ n=1,2,3,\ldots$	$(-1)^n \frac{d^n}{ds^n}F(s)$		

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Fourier Series

Fourier series expansion of periodic function with period $2L$	**Fourier half-range series expansion**
$f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{L}+\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{L}$	$f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{L}+\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{L}$
where	where
$a_0=\frac{1}{L}\int_{-L}^{L}f(x)\,dx$	$a_0=\frac{2}{L}\int_{0}^{L}f(x)\,dx$
$a_n=\frac{1}{L}\int_{-L}^{L}f(x)\cos\frac{n\pi x}{L}\,dx$	$a_n=\frac{2}{L}\int_{0}^{L}f(x)\cos\frac{n\pi x}{L}\,dx$
$b_n=\frac{1}{L}\int_{-L}^{L}f(x)\sin\frac{n\pi x}{L}\,dx$	$b_n=\frac{2}{L}\int_{0}^{L}f(x)\sin\frac{n\pi x}{L}\,dx$

INDEFINITE INTEGRALS

$\int x^n dx=\frac{x^{n+1}}{n+1}+C,\quad n\neq -1$	$\int \csc^2 x\,dx=-\cot x+C$	$\int \cosh x\,dx=\sinh x+C$
$\int \frac{1}{x}dx=\ln\lvert x\rvert+C$	$\int \sec x\tan x\,dx=\sec x+C$	$\int \sinh x\,dx=\cosh x+C$
$\int \cos x\,dx=\sin x+C$	$\int \csc x\cot x\,dx=-\csc x+C$	$\int \text{sech}^2 x\,dx=\tanh x+C$
$\int \sin x\,dx=-\cos x+C$	$\int e^x\,dx=e^x+C$	$\int \text{csch}^2 x\,dx=-\coth x+C$
$\int \sec^2 x\,dx=\tan x+C$		$\int \text{sech}\,x\tanh x\,dx=-\text{sech}\,x+C$
		$\int \text{csch}\,x\coth x\,dx=-\text{csch}\,x+C$

QUADRATIC FORMULA

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

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Formula

DERIVATIVES

$$\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$$

$$\frac{d[u^n]}{dx}=nu^{n-1}\frac{du}{dx}$$

$$\frac{d}{dx}\{f[g(x)]\}=f'[g(x)]g'(x)$$

$$\frac{dy}{dx}=\frac{dy}{du}.\frac{du}{dx}$$

$$\frac{dy}{dx}=\frac{dy}{du}.\frac{du}{dv}.\frac{dv}{dx}$$

IDENTITIES OF TRIGONOMETRY

$\cos^2 x+\sin^2 x=1$ $\sin 2x=2\sin x\cos x$ $\cos 2x=\cos^2 x-\sin^2 x$ $\cos 2x=2\cos^2 x-1$ $\cos 2x=1-2\sin^2 x$	$1+\tan^2 x=\sec^2 x$ $1+\cot^2 x=\csc^2 x$ $\tan 2x=\frac{2\tan x}{1-\tan^2 x}$	$\tan(x\pm y)=\frac{\tan x\pm\tan y}{1\mp\tan x\tan y}$ $\sin(x\pm y)=\sin x\cos y\pm\sin y\cos x$ $\cos(x\pm y)=\cos x\cos y\mp\sin x\sin y$ $2\sin ax\cos bx=\sin(a+b)x+\sin(a-b)x$ $2\sin ax\sin bx=\cos(a-b)x-\cos(a+b)x$ $2\cos ax\cos bx=\cos(a-b)x+\cos(a+b)x$