



UNIVERSITI TUN HUSSEIN ONN MALAYSIA

**FINAL EXAMINATION
SEMESTER I
SESSION 2013/2014**

COURSE NAME : CIVIL ENGINEERING
MATHEMATICS III
COURSE CODE : BFC 24103
PROGRAMME : 2 BFF
EXAMINATION DATE : DECEMBER 2013/JANUARY 2014
DURATION : 3 HOURS
INSTRUCTION : ANSWERS ALL QUESTIONS

THIS QUESTION PAPER CONSISTS OF SIX (6) PAGES

- Q1** (a) Find the values of all relative extrema and saddle points of the function

$$f(x, y) = 4xy - x^4 - y^4 + 10.$$

(10 marks)

- (b) Evaluate $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{x^2+y^2}^{2-x^2-y^2} (x^2 + y^2)^{3/2} dz dy dx$.

(10 marks)

- Q2** (a) The displacement of a particle at time t is given by

$$\mathbf{r}(t) = 3\cos t \mathbf{i} + 3\sin t \mathbf{j} + 4t \mathbf{k}, \quad 0 \leq t \leq 2\pi.$$

- (i) Find the velocity $\mathbf{v}(t)$, the unit tangent vector $\mathbf{T}(t)$, and the unit normal vector $\mathbf{N}(t)$ at $t = \pi$.

(6 marks)

- (ii) Calculate the arc length of $\mathbf{r}(t)$.

(2 marks)

- (iii) Sketch the graph of $\mathbf{r}(t)$.

(2 marks)

- (b) Find the moment of inertia about z -axis, I_z of a solid G with density $\delta(x, y, z) = 4$, where G is the solid bounded by the spheres

$$x^2 + y^2 + z^2 = 4 \quad \text{and} \quad x^2 + y^2 + z^2 = 9.$$

(10 marks)

- Q3** (a) Evaluate the line integral

$$\int_C (x^2 + y^2 - x) dx + y\sqrt{x^2 + y^2} dy$$

where C is the path consists of a line segment from $(0, 0)$ to $(1, 0)$ and along a semicircle $y = \sqrt{1-x^2}$ from $(1, 0)$ to $(-1, 0)$.

(10 marks)

- (b) Given the vector field $\mathbf{F}(x, y, z) = 2xy \mathbf{i} + (x^2 + z^2) \mathbf{j} + 2yz \mathbf{k}$.

- (i) Show that \mathbf{F} is a conservative vector field.

(3 marks)

- (ii) Find its potential function $\phi(x, y, z)$.

(5 marks)

- (iii) Hence, find the work done in this field to move an object from a point $(1, 0, 0)$ to $(0, 1, 2\pi)$.

(2 marks)

- Q4** (a) Use Green's theorem to evaluate the integral

$$\oint_C x^2 y \, dx + (y + xy^2) \, dy,$$

where C is the boundary of the region enclosed by $y = x^2$ and $x = y^2$, and curve C is oriented counterclockwise.

(10 marks)

- (b) Use Stokes' theorem to evaluate the work done by the force field

$$\mathbf{F}(x, y, z) = xz \mathbf{i} + xy^2 \mathbf{j} + 3xz \mathbf{k}$$

along the curve C where C is the intersection between the cylinder $x^2 + y^2 = 4$ and the plane $y + z = 3$ oriented counter clockwise.

(10 marks)

- Q5** (a) Evaluate $\iint_S (1 - x^2 - y^2) \, dS$, where S is the surface of the hemisphere

$x^2 + y^2 + z^2 = 1$ that lies over the xy -plane.

Spherical

(10 marks)

- (b) Let S be the surface of a solid bounded by the paraboloid $z = 4 - x^2 - y^2$ and the xy -plane, oriented by an outwards unit normal vector. Use the Divergence theorem to evaluate

Sym Cylindrical

$$\iint_S \mathbf{F} \cdot \mathbf{n} \, dS,$$

the flux of the vector field $\mathbf{F}(x, y, z) = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$ over the surface S .

(10 marks)

END OF QUESTION -

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Formulae

Polar coordinate: $x = r \cos \theta$, $y = r \sin \theta$, $\theta = \tan^{-1}(y/x)$, and

$$\iint_R f(x, y) dA = \iint_R f(r, \theta) r dr d\theta$$

Cylindrical coordinate: $x = r \cos \theta$, $y = r \sin \theta$, $z = z$,

$$\iiint_G f(x, y, z) dV = \iiint_G f(r, \theta, z) r dz dr d\theta$$

Spherical coordinate: $x = \rho \sin \phi \cos \theta$, $y = \rho \sin \phi \sin \theta$, $z = \rho \cos \phi$,

$$x^2 + y^2 + z^2 = \rho^2,$$

$$0 \leq \theta \leq 2\pi, 0 \leq \phi \leq \pi, \text{ and}$$

$$\iiint_G f(x, y, z) dV = \iiint_G f(\rho, \phi, \theta) \rho^2 \sin \phi d\rho d\phi d\theta$$

Directional derivative: $D_{\mathbf{u}} f(x, y) = (f_x \mathbf{i} + f_y \mathbf{j}) \cdot \mathbf{u}$

Let $\mathbf{F}(x, y, z) = M \mathbf{i} + N \mathbf{j} + P \mathbf{k}$ is vector field, then

$$\text{the divergence of } \mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$$

the curl of

$$\mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z} \right) \mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z} \right) \mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) \mathbf{k}$$

Let C is a smooth curve given by $\mathbf{r}(t) = x(t) \mathbf{i} + y(t) \mathbf{j} + z(t) \mathbf{k}$, t is parameter, then

$$\text{the unit tangent vector: } \mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$$

$$\text{the unit normal vector: } \mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$$

$$\text{the binormal vector: } \mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$$

$$\text{the curvature: } \kappa = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \frac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$$

$$\text{the radius of curvature: } \rho = 1/\kappa$$

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Green Theorem:
$$\oint_C M dx + N dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dA$$

Gauss Theorem:
$$\iint_S \mathbf{F} \cdot \mathbf{n} dS = \iiint_G \nabla \cdot \mathbf{F} dV$$

Stokes' Theorem:
$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} dS$$

Arc length : If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, $t \in [a, b]$, then the arc length

$$s = \int_a^b \|\mathbf{r}'(t)\| dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2} dt$$

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, $t \in [a, b]$, then the arc length

$$s = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2} dt$$

Tangent Plane :

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

Extreme of two variable functions

$$D(x, y) = f_{xx}(x, y)f_{yy}(x, y) - (f_{xy}(x, y))^2$$

Case1: If $D(a, b) > 0$ and $f_{xx}(x, y) < 0$ then f has local maximum at (a, b)

Case2: If $D(a, b) > 0$ and $f_{xx}(x, y) > 0$ then f has local minimum at (a, b)

Case3: If $D(a, b) < 0$ then f has a saddle point at (a, b)

Case4: If $D(a, b) = 0$ then no conclusion can be made.

In 2-D: Lamina

Mass: $m = \iint_R \delta(x, y) dA$, where $\delta(x, y)$ is a density of lamina.

Moment of mass: (i) about y -axis, $M_y = \iint_R x \delta(x, y) dA$,

(ii) about x -axis, $M_x = \iint_R y \delta(x, y) dA$

Centre of mass, $(\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m} \right)$

Moment inertia: (i) $I_y = \iint_R x^2 \delta(x, y) dA$,

(ii) $I_x = \iint_R y^2 \delta(x, y) dA$,

(iii) $I_o = \iint_R (x^2 + y^2) \delta(x, y) dA$

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In 3-D: Solid

Mass, $m = \iiint_G \delta(x, y, z) dV$. If $\delta(x, y, z) = c$, c is a constant, then $m = \iiint_G dA$ is volume.

Moment of mass

- (i) about yz -plane, $M_{yz} = \iiint_G x \delta(x, y, z) dV$
- (ii) about xz -plane, $M_{xz} = \iiint_G y \delta(x, y, z) dV$
- (iii) about xy -plane, $M_{xy} = \iiint_G z \delta(x, y, z) dV$

Centre of gravity, $(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m} \right)$

Moment inertia

- (i) about x -axis: $I_x = \iiint_G (y^2 + z^2) \delta(x, y, z) dV$
- (ii) about y -axis: $I_y = \iiint_G (x^2 + z^2) \delta(x, y, z) dV$
- (iii) about z -axis: $I_z = \iiint_G (x^2 + y^2) \delta(x, y, z) dV$