Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2013/2014

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS II
COURSE CODE	:	BFC 14003
PROGRAMME	:	2 BFF
EXAMINATION DATE	:	DECEMBER 2013/JANUARY 2014
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS IN SECTION A B) ANSWER **TWO (2)** QUESTIONS ONLY IN SECTION B

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

SECTION A

Q1 (a) Determine whether each of the following series converges or diverges by using appropriate test.

(i) $\sum_{n=1}^{\infty} \frac{5}{2n}$.

(2 marks)

(ii) $\sum_{n=1}^{\infty} \frac{3^n}{n!}$.

(3 marks)

(b) Find the interval and radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$.

(11 marks)

(c) Find the Maclaurin series for $\sin x$ up to x^7. Then approximate $\int_1^2 2\sin x^2 \, dx$.

(9 marks)

Q2 A periodic function is defined by

$$f(x) = \begin{cases} -2x, & -1 < x < 0 \\ 2x, & 0 < x < 1 \end{cases}$$
$$f(x) = f(x+2).$$

(a) Sketch the graph of the function over $-3 < x < 3$.

(4 marks)

(b) Determine whether the function is even, odd or neither.

(2 marks)

(c) Show that the Fourier series of the function $f(x)$ is

$$1 + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \left(\frac{\cos(n\pi) - 1}{n^2} \right) \cos(n\pi x).$$

(19 marks)

SECTION B

Q3 (a) Given first order ordinary differential equation

$$\frac{dy}{dx}=\frac{x^3+y^3}{xy^2}.$$

(i) Show that the equation is homogeneous.

(3 marks)

(ii) Hence, find its general solution.

(8 marks)

(b) Find the solution of initial value problem

$$x^3y'+4x^2y=e^{-x}$$

where $y(0)=-1$. Use the method of linear equation.

(14 marks)

Q4 (a) Find the general solution of homogeneous second order differential equation

$$9\frac{d^2y}{dx^2}-6\frac{dy}{dx}+16y=0.$$

(8 marks)

(b) Given

$$y''-2y'+y=\frac{e^x}{1+x^2}.$$

(i) Show that $y_1=e^x$ and $y_2=xe^x$.

(3 marks)

(ii) Hence, by using the method variation of parameters, find the general solution of the differential equation.

$\left[\text{Hint}: \int\frac{1}{1+x^2}\,dx=\tan^{-1}x+C.\right]$

(14 marks)

Q5 (a) Find the Laplace transform of the given functions.

(i) $f(t) = t^3 + \cosh\sqrt{3}t$.

(1 marks)

(ii) $f(t) = t\sin 2t$.

(2 marks)

(b) Find the inverse Laplace transform of

$$F(s) = \frac{2}{s^4} + \frac{3s+1}{s^2 - 4s + 20}.$$

(8 marks)

(c) Solve initial value problem

$$y'' + 3y' + 2y = H(t-3), \quad y(0) = 0, \quad y'(0) = 0$$

by using Laplace transform.

(14 marks)

- END OF QUESTION -

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FORMULA

The roots of characteristic equation and the general solution for differential equation $ay'' + by' + cy = 0$.

Characteristic equation: $am^2 + bm + c = 0$.		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y = Ae^{m_1 x} + Be^{m_2 x}$
2.	Real and equal roots: $m = m_1 = m_2$	$y = (A + Bx)e^{mx}$
3.	Complex roots: $m_1 = \alpha + \beta i$, $m_2 = \alpha - \beta i$	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

The method of undetermined coefficients

For non-homogeneous second order differential equation $ay'' + by' + cy = f(x)$, the particular solution is given by $y_p(x)$:

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + A_{n-1}x^{n-1} + \cdots + A_1 x + A_0$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(P\cos\beta x + Q\sin\beta x)$
$P_n(x)e^{\alpha x}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)e^{\alpha x}$
$P_n(x)\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)\cos\beta x +$ $x^r(C_n x^n + C_{n-1}x^{n-1} + \cdots + C_1 x + C_0)\sin\beta x$
$Ce^{\alpha x}\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^r e^{\alpha x}(P\cos\beta x + Q\sin\beta x)$
$P_n(x)e^{\alpha x}\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)e^{\alpha x}\cos\beta x +$ $x^r(C_n x^n + C_{n-1}x^{n-1} + \cdots + C_1 x + C_0)e^{\alpha x}\sin\beta x$

Note : r is the least non-negative integer (r = 0, 1, or 2) which determine such that there is no terms in particular integral $y_p(x)$ corresponds to the complementary function $y_c(x)$.

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The method of variation of parameters

If the solution of the homogeneous equation $ay'' + by' + cy = 0$ is $y_c = Ay_1 + By_2$, then the particular solution for $ay'' + by' + cy = f(x)$ is

$$y = uy_1 + vy_2,$$

where $u = -\int \frac{y_2 f(x)}{aW} dx \quad v = \int \frac{y_1 f(x)}{aW} dx$ with $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$.

Table of Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
e^{at}	$\frac{1}{s-a}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\sin at$	$\frac{a}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\cos at$	$\frac{s}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$y(t)$	$Y(s)$
t^n, $n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$	$y'(t)$	$sY(s) - y(0)$
$e^{at}f(t)$	$F(s-a)$	$y''(t)$	$s^2Y(s) - sy(0) - y'(0)$
$t^n f(t)$, $n = 1, 2, 3, \ldots$	$(-1)^n \frac{d^n}{ds^n}F(s)$		

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Taylor Series

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

Maclaurin Series

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

Fourier Series

Fourier series expansion of periodic function with period $2L$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$

where

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\, dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\, dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\, dx$$

Fourier half-range series expansion

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$

where

$$a_0 = \frac{2}{L}\int_{0}^{L} f(x)\, dx$$

$$a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L}\, dx$$

$$b_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L}\, dx$$