Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2013/2014

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS 1
COURSE CODE	:	BFC 13903
PROGRAMME	:	1 BFF
EXAMINATION DATE	:	DECEMBER 2013/JANUARY 2014
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS IN SECTION A B) ANSWER **FOUR** (4) QUESTIONS IN SECTION B

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

SECTION A

Q1 (a) By using $\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$, find the derivative of the $f(x)$ with respect to x if $f(x) = \ln\left[\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)\right]$.

(4 marks)

(b) Evaluate the given integral

$$\int \frac{1+6x}{\sqrt{4+9x^2}}\,dx \text{ given that } \int \frac{1}{\sqrt{a^2+x^2}}\,dx = \sinh^{-1}\left(\frac{x}{a}\right) + C.$$

(5 marks)

(c) The parametric equation of a semi-circle is $x = a\cos\theta$ and $y = a\sin\theta$, where $0 \le \theta \le \pi$. Find the surface area generated by rotating the semi-circle about the x-axis through 360°.

(5 marks)

(d) Find the curvature (κ) of the given curve, $y = \dfrac{x-1}{x^2+3}$ at $x = 0$.

(6 marks)

SECTION B

Q2 (a) A number of workers p is required to produce q units of a certain goods. The relationship between p and q is given by $q = 16p^2$. If the current production is 160 000 units per year and increase at a rate of 32 000 units per year, what is the rate of increase in the number of workers?

(4 marks)

(b) Given a curve $f(x) = \dfrac{x}{(x-1)^2}$.

(i) Find x-intercept and y-intercept.

(ii) Show that $f(x)$ has a vertical asymptote at $x = 1$ and a horizontal asymptote at x-axis.

(iii) Find the critical points of $f(x)$. Determine the extremum point.

(iv) If $f(x)$ has inflection point at $x = -2$, sketch the graph of $f(x)$. Show all the asymptote (s), intersection point (s), extremum and inflection point (s) if there is any in your graph.

(8 marks)

(c) Use *L'hôpital* rule to find the limits below.

(i) $\lim_{x \to 1} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right)$,

(ii) $\lim_{x \to 0} (\sin 2x + 1)^{\frac{1}{x}}$.

(8 marks)

Q3 (a) Find $\frac{dy}{dx}$ if $y = x^2 \sin(3x) + 2^x$.

(5 marks)

(b) (i) By using the Chain Rule, find the slope of the curve $x = \frac{1}{1-t^2}$ and $y = \frac{1}{1+t^2}$ when $t = 2$.

(ii) Then, find $\frac{d^2y}{dx^2}$.

(10 marks)

(c) By using the Implicit Differentiation, find $\frac{dy}{dx}$ for $\ln y + 2xy^2 = 10 - x$.

(5 marks)

Q4 (a) Let

$$f(x) = \begin{cases} 5 & , \quad x < 2 \\ x^2 + k & , \quad 2 \le x < 3 \\ \dfrac{x^2 - mx - 6}{x - 2} & , \quad x \ge 3 \end{cases}$$

Determine the value of constants k and m for which $f(x)$ is continuous for all values of x.

(6 marks)

(b) Differentiate $y = x\sqrt{3x^2+6}$ with respect to x. Hence, evaluate

$$\int_1^5 \frac{x^2+1}{2\sqrt{3x^2+6}}\,dx\,.$$

(6 marks)

(c) Find the first derivative for the following inverse trigonometric functions.

(i) $y = \tan^{-1}(\sin x)$,

(ii) $y = (\cos^{-1} x)^3$.

(8 marks)

Q5 (a) Use the given substitution to evaluate the indicated integral

$$\int x^2\left(\sqrt{x^3+2}\right)dx \quad , \quad u = x^3+2\,.$$

(4 marks)

(b) Analyze and evaluate the following integral.

$$\int_{\frac{\pi}{2}}^{\pi} \cos^3 x\,dx\,.$$

(5 marks)

(c) Find $\int (\sin 4x - \cos 3x)^2\,dx$.

(5 marks)

(d) Evaluate the following integral

$$\int_0^1 \sqrt{16-5x^2}\,dx\,.$$

(6 marks)

Q6 (a) By using suitable techniques, evaluate each of the following integrals.

(i) $\displaystyle\int \frac{10x}{(x-2)^3}\,dx$,

(ii) $\int e^{2x}\sin 3x\,dx$.

(9 marks)

(b) Evaluate the following integral by using hyperbolic substitution

$$\int \frac{1}{\sqrt{x^2+2x+5}}\,dx.$$

(6 marks)

(c) Evaluate the following integral

$$\int_{\frac{1}{8}}^{\frac{1}{4}} \frac{1}{\sqrt{1-16x^2}}\,dx.$$

(5 marks)

- END OF QUESTION -

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Formulae

TRIGONOMETRIC IDENTITY

$$\sin^2 x + \cos^2 x = 1$$

WEIERSTRASS SUBSTITUTION

$t = \tan \frac{1}{2} x$		$t = \tan x$	
$\sin x = \frac{2t}{1+t^2}$ $\tan x = \frac{2t}{1-t^2}$	$\cos x = \frac{1-t^2}{1+t^2}$ $dx = \frac{2dt}{1+t^2}$	$\sin 2x = \frac{2t}{1+t^2}$ $\tan 2x = \frac{2t}{1-t^2}$	$\cos 2x = \frac{1-t^2}{1+t^2}$ $dx = \frac{dt}{1+t^2}$

IDENTITIES OF TRIGONOMETRY

$\cos^2 x + \sin^2 x = 1$ $\sin 2x = 2\sin x \cos x$ $\cos 2x = \cos^2 x - \sin^2 x$ $\cos 2x = 2\cos^2 x - 1$ $\cos 2x = 1 - 2\sin^2 x$	$1 + \tan^2 x = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$ $\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$	$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$ $\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$ $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$ $2\sin ax \cos bx = \sin(a+b)x + \sin(a-b)x$ $2\sin ax \sin bx = \cos(a-b)x - \cos(a+b)x$ $2\cos ax \cos bx = \cos(a-b)x + \cos(a+b)x$

IDENTITIES OF HYPERBOLIC FUNCTIONS

$\sinh x = \frac{e^x - e^{-x}}{2}$ $\cosh x = \frac{e^x + e^{-x}}{2}$ $\cosh^2 x - \sinh^2 x = 1$ $\sinh 2x = 2\sinh x \cosh x$ $\cosh 2x = \cosh^2 x + \sinh^{2x}$	$\cosh 2x = 2\cosh^2 x - 1$ $\cosh 2x = 1 + 2\sinh^2 x$ $1 - \tanh^2 x = \text{sech}^2 x$ $\coth^2 x - 1 = \text{csch}^2 x$ $\tanh 2x = \frac{2\tanh x}{1 + \tanh^2 x}$	$\tanh(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$ $\sinh(x \pm y) = \sin x \cos y \pm \sin y \cos x$ $\cosh(x \pm y) = \cos x \cos y \pm \sin x \sin y$

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CURVATURE, ARC LENGTH AND SURFACE AREA OF REVOLUTION

$$\kappa = \frac{\left|\dfrac{d^2y}{dx^2}\right|}{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}$$

$$\kappa = \frac{\left|\dot{x}\ddot{y}-\dot{y}\ddot{x}\right|}{\left[\dot{x}^2+\dot{y}^2\right]^{\frac{3}{2}}}$$

$$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

$$L = \int_{x_1}^{x_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$L = \int_{y_1}^{y_2} \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$S = 2\pi\int_{x_1}^{x_2} f(x)\sqrt{1+\left(\frac{d}{dx}[f(x)]\right)^2}\,dx$$

$$S = 2\pi\int_{y_1}^{y_2} g(y)\sqrt{1+\left(\frac{d}{dy}[g(y)]\right)^2}\,dy$$

INDEFINITE INTEGRALS

$$\int x^n dx = \frac{x^{n+1}}{n+1}+C, \quad n \neq -1$$

$$\int \frac{1}{x}dx = \ln|x|+C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \csc x \cot x\,dx = -\csc x + C$$

$$\int e^x\,dx = e^x + C$$

$$\int \cosh x\,dx = \sinh x + C$$

$$\int \sinh x\,dx = \cosh x + C$$

$$\int \text{sech}^2\, x\,dx = \tanh x + C$$

$$\int \text{csch}^2 x\,dx = -\coth x + C$$

$$\int \text{sech}\, x \tanh x\,dx = -\text{sech}\, x + C$$

$$\int \text{csch}\, x \coth x\,dx = -\text{csch}\, x + C$$

TRIGONOMETRIC SUBSTITUTION

Expression	Trigonometry	Hyperbolic
$\sqrt{x^2+k^2}$	$x = k\tan\theta$	$x = k\sinh\theta$
$\sqrt{x^2-k^2}$	$x = k\sec\theta$	$x = k\cosh\theta$
$\sqrt{k^2-x^2}$	$x = k\sin\theta$	$x = k\tanh\theta$

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INTEGRATION OF INVERSE FUNCTIONS

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C, \quad |x|<1$$

$$\int \frac{-1}{\sqrt{1-x^2}}\,dx = \cos^{-1}x + C, \quad |x|<1$$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1}x + C$$

$$\int \frac{-1}{1+x^2}\,dx = \cot^{-1}x + C$$

$$\int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \sec^{-1}x + C, \quad |x|>1$$

$$\int \frac{-1}{|x|\sqrt{x^2-1}}\,dx = \csc^{-1}x + C, \quad |x|>1$$

$$\int \frac{1}{\sqrt{x^2+1}}\,dx = \sinh^{-1}x + C$$

$$\int \frac{1}{\sqrt{x^2-1}}\,dx = \cosh^{-1}x + C, \quad |x|>1$$

$$\int \frac{-1}{|x|\sqrt{1-x^2}}\,dx = \operatorname{sech}^{-1}|x| + C, \quad 0<x<1$$

$$\int \frac{-1}{|x|\sqrt{1+x^2}}\,dx = \operatorname{csch}^{-1}|x| + C, \quad x\neq 0$$

$$\int \frac{1}{1-x^2}\,dx = \tanh^{-1}x + C, \quad |x|<1$$

$$\int \frac{1}{1-x^2}\,dx = \coth^{-1}x + C, \quad |x|>1$$

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DERIVATIVES

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{d\left[u^n\right]}{dx} = nu^{n-1}\frac{du}{dx}$$

$$\frac{d}{dx}\{f[g(x)]\} = f'[g(x)]g'(x)$$

$$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dv}.\frac{dv}{dx}$$

QUADRATIC FORMULA

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$