SET A

CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2012/2013

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS II
COURSE CODE	:	BFC 14003
PROGRAMME	:	1 BFF / 2 BFF
EXAMINATION DATE	:	JUNE 2013
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B**

THIS QUESTION PAPER CONSISTS OF SIX (6) PAGES

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PART A

Q1 A periodic function is defined by

$$f(x)=\begin{cases}2\pi, & -\pi<x<0\\ 2x, & 0<x<\pi\end{cases}$$
$$f(x)=f(x+2\pi).$$

(a) Sketch the graph of the function over $-\pi<x<\pi$.

(2 marks)

(b) Determine whether the function is even, odd or neither.

(1 mark)

(c) Show that the Fourier series of the function $f(x)$ is

$$\frac{3\pi}{2}+2\sum_{n=1}^{\infty}\frac{\left[(-1)^n-1\right]}{\pi n^2}\cos(nx)-2\sum_{n=1}^{\infty}\frac{1}{n}\sin(nx).$$

(17 marks)

Q2 (a) Given that $\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\ldots$.

(i) Find the first three nonzero terms of a power series for $\frac{\sin x}{\sqrt{x}}$.

(3 marks)

(ii) Hence, evaluate $\int_0^1 \frac{\sin x}{\sqrt{x}}\,dx$ by using the series expansion.

(3 marks)

(b) (i) Determine whether the series $\sum_{n=1}^{\infty}\frac{n^2}{e^n}$ converges or diverges by using ratio test.

(3 marks)

(ii) Determine whether the series $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ converges absolutely, converges conditionally, or diverges by using a suitable convergence test.

(5 marks)

(c) Find the radius of convergence of $\sum_{n=0}^{\infty}3(x-2)^n$.

(6 marks)

PART B

Q3 (a) Solve the differential equation given below by using the method of separation of variables,

$$(1+x)\frac{dy}{dx} = x\sqrt{1-y}.$$

Hence, find the particular solution when $y(1) = 0$.

(7 marks)

(b) By using the substitution of $y = xv$ and $\frac{dy}{dx} = x\frac{dv}{dx} + v$, find the solution of

$$\frac{dy}{dx} = \frac{x^2 + y^2}{3xy}.$$

(6 marks)

(c) In a certain culture of bacteria, the rate of increase is proportional to the number of present. If it is found that the number doubles in 4 hours, how many may expected at the end of 12 hours?

[Hint: $\frac{dN}{dt} = kN$, where N denotes the number of bacteria at time, t hours and k is the proportionality factor.]

(7 marks)

Q4 (a) Use the method of variation of parameters to solve

$$y'' + y = \sec x \tan x,$$

which satisfies the initial conditions $y(0) = 0$ and $y'(0) = 2$.

[Hint: $\sec^2 x + \tan^2 x = 1$, $\int \sec^2 x \, dx = \tan x$.]

(10 marks)

(b) A spring is stretched 0.1 m $(=\Delta l)$ when a 4 kg mass $(=M)$ is attached. The weight is then pulled down an additional 0.2 m and released with an upward velocity of 4 m/s. Neglect damping, c. If the general equation describing the spring-mass system is

$$Mu'' + cu' + ku = 0,$$

find an equation for the position of the spring at any time t.

[Hint: weight, $W = Mg$, $g \approx 9.8$, $k = \dfrac{W}{\Delta l}$.]

(10 marks)

Q5 (a) Find

(i) $\mathcal{L}\left\{e^{3t}t^2 + \dfrac{4}{e^{5t}}\right\}$.

(3 marks)

(ii) $\mathcal{L}\left\{t\cosh 4t + t^3\delta(t-3)\right\}$.

(3 marks)

(iii) $\mathcal{L}\left\{\sinh(3t) + e^{3t}H(t-3)\right\}$.

(4 marks)

(b) By using Laplace transform, solve

$$y'' - 2y' + y = e^t, \qquad y(0) = -2,\ y'(0) = -3.$$

(10 marks)

Q6 (a) Find

(i) $\mathcal{L}^{-1}\left\{\dfrac{5s}{s^2-4} + \dfrac{6}{(s-2)^2+4}\right\}$.

(3 marks)

(ii) $\mathcal{L}^{-1}\left\{\dfrac{16s^2}{(s-3)(s+1)^2}\right\}$.

(7 marks)

(b) Find the general solution for the second order differential equation

$$y'' - 3y' + 2y = x^2 + 2 - 5\sin x,$$

by using the undetermined coefficient method.

(10 marks)

-END OF QUESTION-

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FORMULA

Second-order Differential Equation

The roots of characteristic equation and the general solution for differential equation $ay''+by'+cy=0$.

Characteristic equation: $am^2+bm+c=0$.		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y=Ae^{m_1x}+Be^{m_2x}$
2.	Real and equal roots: $m=m_1=m_2$	$y=(A+Bx)e^{mx}$
3.	Complex roots: $m_1=\alpha+\beta i$, $m_2=\alpha-\beta i$	$y=e^{\alpha x}(A\cos\beta x+B\sin\beta x)$

The method of undetermined coefficients

For non-homogeneous second order differential equation $ay''+by'+cy=f(x)$, the particular solution is given by $y_p(x)$:

$f(x)$	$y_p(x)$
$P_n(x)=A_nx^n+A_{n-1}x^{n-1}+\cdots+A_1x+A_0$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(P\cos\beta x+Q\sin\beta x)$
$P_n(x)e^{\alpha x}$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)e^{\alpha x}$
$P_n(x)\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)\cos\beta x+$ $x^r(C_nx^n+C_{n-1}x^{n-1}+\cdots+C_1x+C_0)\sin\beta x$
$Ce^{\alpha x}\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^re^{\alpha x}(P\cos\beta x+Q\sin\beta x)$
$P_n(x)e^{\alpha x}\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)e^{\alpha x}\cos\beta x+$ $x^r(C_nx^n+C_{n-1}x^{n-1}+\cdots+C_1x+C_0)e^{\alpha x}\sin\beta x$

Note : r is the least non-negative integer (r = 0, 1, or 2) which determine such that there is no terms in particular integral $y_p(x)$ corresponds to the complementary function $y_c(x)$.

The method of variation of parameters

If the solution of the homogeneous equation $ay''+by'+cy=0$ is $y_c=Ay_1+By_2$, then the particular solution for $ay''+by'+cy=f(x)$ is

$$y=uy_1+vy_2,$$

where $u=-\int\frac{y_2f(x)}{aW}dx+A$, $v=\int\frac{y_1f(x)}{aW}dx+B$ and $W=\begin{vmatrix}y_1 & y_2\\ y_1' & y_2'\end{vmatrix}=y_1y_2'-y_2y_1'$.

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Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
e^{at}	$\frac{1}{s-a}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\sin at$	$\frac{a}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\cos at$	$\frac{s}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$y(t)$	$Y(s)$
$t^n,\ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$y'(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$
$t^n f(t),\ n=1,2,3,\ldots$	$(-1)^n \frac{d^n}{ds^n}F(s)$		

Fourier Series

Fourier series expansion of periodic function with period $2L$	**Fourier half-range series expansion**
$f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{L}+\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{L}$	$f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{L}+\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{L}$
where	where
$a_0=\frac{1}{L}\int_{-L}^{L}f(x)\,dx$	$a_0=\frac{2}{L}\int_0^L f(x)\,dx$
$a_n=\frac{1}{L}\int_{-L}^{L}f(x)\cos\frac{n\pi x}{L}\,dx$	$a_n=\frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx$
$b_n=\frac{1}{L}\int_{-L}^{L}f(x)\sin\frac{n\pi x}{L}\,dx$	$b_n=\frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx$