UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2012/2013

COURSE NAME	:	ENGINEERING MATHEMATICS 1
COURSE CODE	:	BFC 13903
PROGRAMME	:	1 BFF
EXAMINATION DATE	:	DECEMBER 2012/JANUARY 2013
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** IN **SECTION A** AND **FOUR (4)** IN **SECTION B**

THIS QUESTION PAPER CONSISTS OF **FOURTEEN (14)** PAGES

SECTION A

Q1 (a) Find the first derivative for the following inverse functions:

(i) $f(x) = \sin^{-1}(x^2 - 1)$.

(2 marks)

(ii) $f(x) = \tanh^{-1}(\tanh^{-1}(2x))$.

(3 marks)

(b) Show that $\int \sin^{-1} x \, dx = x \sin^{-1} x + \sqrt{1-x^2} + C$.

(7 marks)

(c) Evaluate $\int \frac{1+2x}{\sqrt{9+4x}} dx$ given that $\int \frac{1}{\sqrt{a^2+x^2}} dx = \sinh^{-1}\left(\frac{x}{a}\right) + C$.

(8 marks)

SECTION B

Q2 (a) For the curve $y = 2x^3 - 11x^2 + 12x + 9$, determine:

(i) The values of x so that $\frac{dy}{dx} = 0$.

(ii) The values of y so that $\frac{dy}{dx} = 0$.

(3 marks)

(b) The railway is connecting Town A, Town B and C as shown in **Figure Q2(b).** Determine the branch angle θ which can minimize the overall construction costs of the railway. Assume construction cost from;

Town A to Town B = RM 50000 per kilometer

Town B to Town C = RM 100000 per kilometer

(6 marks)

(c) Given a curve $y = x^5 + 5x^3 - 20x + 3$.

(i) Find the critical points.

(ii) Determine the point of inflection.

(iii) Fill the blanks in **Table 1**.

(iv) Sketch the graph of $y = x^5 + 5x^3 - 20x + 3$.

(11 marks)

Q3 (a) By using the **First Principle**, find $f'(x)$ if $f(x) = 4x^2 - 5x$.

(3 marks)

(b) By using the **Product Rule**, find $f(x) = (2x^4 - 3x + 5)\left(x^2 - \sqrt{x} + \frac{2}{x}\right)$.

(3 marks)

(c) By using the **Quotient Rule**, find $f'(x)$ of $f(x) = \frac{\sqrt{x} - 1}{\sqrt{x} + 1}$.

(4 marks)

(d) By using the **Chain Rule**, find the following:

(i) $\frac{du}{dv}$ if $u = t + \frac{1}{t}$ and $t = 1 - \frac{1}{v}$.

(ii) $\frac{dr}{dt}$ if $r = (3s + 1)^{\frac{1}{2}}$ and $s = 16t^2 - 20t$.

(5 marks)

(e) Solves for the following parts, by using the **Implicit Differentation**:

(i) Find $\frac{dy}{dx}$ for $x^2 + y^3 = 2y + 3$ at $x = 2$ and $y = 1$.

(ii) Find $\frac{dy}{dx}$ for $x\sqrt{x + y} = 2xy^2$.

(5 marks)

Q4 (a) Find

(i) $\lim_{x\to 0}\dfrac{20\pi}{x-1}$

(ii) $\lim_{x\to -6}\dfrac{\frac{x}{2}+3}{|6+x|}$

(4 marks)

(b) Compute

(i) $\lim_{x\to 4}\dfrac{3(x-4)\sqrt{x+5}}{3-\sqrt{x+5}}$

(ii) $\lim_{x\to\infty}\sqrt{\dfrac{e^x}{e^x+2}}$

(7 marks)

(c) Let

$$f(x)=\begin{cases}\dfrac{\cos^2 x+3\cos x+k}{7} & ,x<\pi\\ 16-m & ,x=\pi\\ \dfrac{\sin\left(\dfrac{3x}{2}\right)-10k}{\cos 2x} & ,x>\pi\end{cases}.$$

Determine the value of constants k and m for which $f(x)$ is continuous at $x=\pi$.

(9 marks)

Q5 (a) If $f(x)$ is continuous and non-negative on the interval $[a,b]$ with c located between a and b, sketch and show that $\int_a^b f(x)dx=\int_a^c f(x)dx+\int_c^b f(x)dx$.

(5 marks)

(b) Solve the following with the integration by **Substitution Method**.

(i) $\int\dfrac{1}{\left(\frac{x}{3}-8\right)^5}dx$

(2 marks)

(ii) $\int\sin 5x\,dx$

(2 marks)

(b) (iii) $\int t^4 \sqrt[3]{3-5t^5}\,dt$

(3 marks)

(c) Analyse and evaluate the following integrals **Trigonometric Functions**.

(i) $\int_{\frac{\pi}{2}}^{\pi} \cos^3 x\,dx$

(4 marks)

(ii) $\int_{0}^{\frac{\pi}{2}} \cos^5 x\,dx$

(4 marks)

Q6 Solve to an integrals involving sine and cosine of a **Rational Functions**.

(a) $\int \frac{dx}{\sin x}$

(6 marks)

(b) $\int \frac{5}{4+\sin x}dx$

(7 marks)

(c) $\int \frac{dx}{3\sin x+4\cos x+4}$

(7 marks)

BAHAGIAN A

S1 (a) Dapatkan pembezaan pertama bagi fungsi songsangan yang berikut:

(i) $f(x) = \sin^{-1}(x^2 - 1)$.

(ii) $f(x) = \tanh^{-1}(\tanh^{-1}(2x))$.

(5 markah)

(b) Buktikan $\int \sin^{-1} x \, dx = x \sin^{-1} x + \sqrt{1-x^2} + C$.

(7 markah)

(c) Nilaikan $\int \frac{1+2x}{\sqrt{9+4x}} dx$ yang mana $\int \frac{1}{\sqrt{a^2+x^2}} dx = \sinh^{-1}\left(\frac{x}{a}\right) + C$.

(8 markah)

BAHAGIAN B

S2 (a) Bagi satu lengkuk $y = 2x^3 - 11x^2 + 12x + 9$, tentukan:

(i) Nilai-nilai bagi x bagi $\frac{dy}{dx} = 0$.

(ii) Nilai-nilai bagi y bagi $\frac{dy}{dx} = 0$.

(3 markah)

(b) Satu landasan keretapi menghubungkan Bandar A, Bandar B dan Bandar C sepertimana yang ditunjukkan dalam **Rajah Q2(b).** Tentukan sudut θ yang mana ianya boleh meminimakan keseluruhan kos pembinaan bagi landasan keretapi tersebut. Anggapkan kos pembinaan daripada:

Bandar A ke Bandar B = RM 50000 per kilometer

Bandar B ke Bandar C = RM 100000 per kilometer

(6 markah)

(c) Diberikan satu lengkuk $y = x^5 + 5x^3 - 20x + 3$.

(i) Dapatkan titik kritikal.

(ii) Tentukan titik lengkuk balas.

(iii) Penuhkan **Jadual 1**.

(iv) Lakrkan graf $y = x^5 + 5x^3 - 20x + 3$.

(11 markah)

S3 (a) Dengan menggunakan **Prinsip Pertama**, dapatkan $f'(x)$ jika $f(x) = 4x^2 - 5x$.

(3 markah)

(b) Dengan menggunakan **Petua *Product***, dapatkan

$$f(x) = (2x^4 - 3x + 5)\left(x^2 - \sqrt{x} + \frac{2}{x}\right).$$

(3 markah)

(c) Dengan menggunakan **Petua *Quotient***, dapatkan $f'(x)$ bagi

$$f(x) = \frac{\sqrt{x} - 1}{\sqrt{x} + 1}.$$

(4 markah)

(d) Dengan menggunakan **Petua Rantai**, tentukan yang berikut:

(i) $\frac{du}{dv}$ jika $u = t + \frac{1}{t}$ dan $t = 1 - \frac{1}{v}$.

(ii) $\frac{dr}{dt}$ jika $r = (3s+1)^{\frac{1}{2}}$ dan $s = 16t^2 - 20t$.

(5 markah)

(e) Selesaikan fungsi bagi bahagian berikut, dengan menggunakan **Pembezaan Tersirat**, bagi yang berikut:

(i) $\frac{dy}{dx}$ bagi $x^2 + y^3 = 2y + 3$ at $x = 2$ dan $y = 1$.

(ii) $\frac{dy}{dx}$ bagi $x\sqrt{x+y} = 2xy^2$.

(5 markah)

S4 (a) Tentukan yang berikut:

(i) $\lim\limits_{x\to 0} \dfrac{20\pi}{x-1}$

(ii) $\lim\limits_{x\to -6} \dfrac{\frac{x}{2}+3}{|6+x|}$

(4 markah)

(b) Kirakan:

(i) $\lim\limits_{x\to 4} \dfrac{3(x-4)\sqrt{x+5}}{3-\sqrt{x+5}}$

(ii) $\lim\limits_{x\to \infty} \sqrt{\dfrac{e^x}{e^x+2}}$

(7 markah)

(c) Jika

$$f(x)=\begin{cases} \dfrac{\cos^2 x+3\cos x+k}{7} & ,x<\pi \\ 16-m & ,x=\pi \\ \dfrac{\sin\left(\frac{3x}{2}\right)-10k}{\cos 2x} & ,x>\pi \end{cases}.$$

Tentukan pemalar k dan m bagi $f(x)$ yang berterusan atau selanjar pada $x=\pi$.

(9 markah)

S5 (a) Jika fungsi $f(x)$ merupakan selanjar dan bukan negative pada selang is $[a,b]$ dengan c terletak di antara a dan b, lakar dan buktikan $\int_a^b f(x)dx=\int_a^c f(x)dx+\int_c^b f(x)dx$.

(5 markah)

(b) Selesaikan fungsi pengamiran yang berikut dengan menggunakan **Kaedah Penggantian.**

(i) $\int \frac{1}{\left(\frac{x}{3}-8\right)^5} dx$

(2 markah)

(ii) $\int \sin 5x \, dx$

(2 markah)

(iii) $\int t^4 \sqrt[3]{3-5t^5} \, dt$

(3 markah)

(c) Analisa dan nilaikan fungsi **Pengamiran Trigonometri** berikut:

(i) $\int_{\frac{\pi}{2}}^{\pi} \cos^3 x \, dx$

(3 markah)

(ii) $\int_{0}^{\frac{\pi}{2}} \cos^5 x \, dx$

(4 markah)

Q6 Selesaikan terhadap kamiran-kamiran yang melibatkan **Fungsi Rasional** sinus dan kosinus.

(a) $\int \frac{dx}{\sin x}$

(6 markah)

(b) $\int \frac{5}{4+\sin x} dx$

(7 markah)

(c) $\int \frac{dx}{3\sin x + 4\cos x + 4}$

(7 markah)

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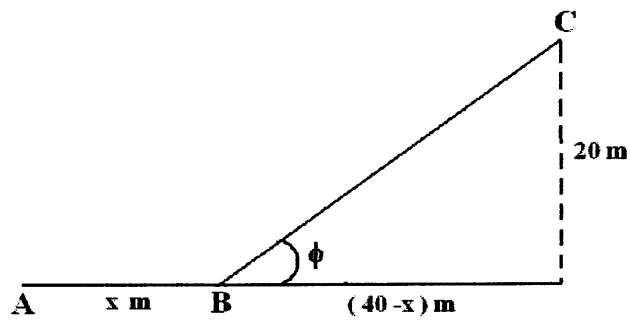

FIGURE Q2(b)

Table 1

Values of x			
Sign of $f'(x)$			
Slope sketching			
Sign of $f''(x)$			
Shape of graph			

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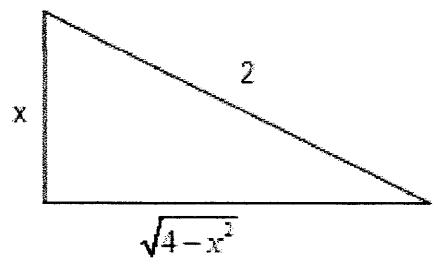

FIGURE Q6(a)

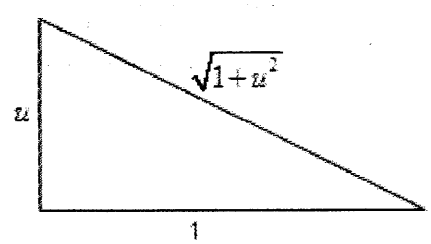

FIGURE Q6(b)

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Formulae

TRIGONOMETRIC IDENTITY

$$\sin^2 x + \cos^2 x = 1$$

WEIERSTRASS SUBSTITUTION

$t = \tan\frac{1}{2}x$		$t = \tan x$	
$\sin x = \frac{2t}{1+t^2}$	$\cos x = \frac{1-t^2}{1+t^2}$	$\sin 2x = \frac{2t}{1+t^2}$	$\cos 2x = \frac{1-t^2}{1+t^2}$
$\tan x = \frac{2t}{1-t^2}$	$dx = \frac{2dt}{1+t^2}$	$\tan 2x = \frac{2t}{1-t^2}$	$dx = \frac{dt}{1+t^2}$

IDENTITIES OF TRIGONOMETRY

$\cos^2 x + \sin^2 x = 1$	$1 + \tan^2 x = \sec^2 x$	$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$
$\sin 2x = 2\sin x \cos x$	$1 + \cot^2 x = \csc^2 x$	$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$
$\cos 2x = \cos^2 x - \sin^2 x$	$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$	$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
$\cos 2x = 2\cos^2 x - 1$		$2\sin ax \cos bx = \sin(a+b)x + \sin(a-b)x$
$\cos 2x = 1 - 2\sin^2 x$		$2\sin ax \sin bx = \cos(a-b)x - \cos(a+b)x$
		$2\cos ax \cos bx = \cos(a-b)x + \cos(a+b)x$

IDENTITIES OF HYPERBOLIC FUNCTIONS

$\sinh x = \frac{e^x - e^{-x}}{2}$	$\cosh 2x = 2\cosh^2 x - 1$	$\tanh(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$
$\cosh x = \frac{e^x + e^{-x}}{2}$	$\cosh 2x = 1 + 2\sinh^2 x$	$\sinh(x \pm y) = \sin x \cos y \pm \sin y \cos x$
$\cosh^2 x - \sinh^2 x = 1$	$1 - \tanh^2 x = \operatorname{sech}^2 x$	$\cosh(x \pm y) = \cos x \cos y \pm \sin x \sin y$
$\sinh 2x = 2\sinh x \cosh x$	$\coth^2 x - 1 = \operatorname{csch}^2 x$	
$\cosh 2x = \cosh^2 x + \sinh^{2x}$	$\tanh 2x = \frac{2\tanh x}{1+\tanh^2 x}$	

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Formulae

INTEGRATION OF INVERSE FUNCTIONS

$$\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}x + C, \quad |x| < 1$$

$$\int \frac{-1}{\sqrt{1-x^2}}dx = \cos^{-1}x + C, \quad |x| < 1$$

$$\int \frac{1}{1+x^2}dx = \tan^{-1}x + C$$

$$\int \frac{-1}{1+x^2}dx = \cot^{-1}x + C$$

$$\int \frac{1}{|x|\sqrt{x^2-1}}dx = \sec^{-1}x + C, \quad |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{x^2-1}}dx = \csc^{-1}x + C, \quad |x| > 1$$

$$\int \frac{1}{\sqrt{x^2+1}}dx = \sinh^{-1}x + C$$

$$\int \frac{1}{\sqrt{x^2-1}}dx = \cosh^{-1}x + C, \quad |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{1-x^2}}dx = \operatorname{sech}^{-1}|x| + C, \quad 0 < x < 1$$

$$\int \frac{-1}{|x|\sqrt{1+x^2}}dx = \operatorname{csch}^{-1}|x| + C, \quad x \neq 0$$

$$\int \frac{1}{1-x^2}dx = \tanh^{-1}x + C, \quad |x| < 1$$

$$\int \frac{1}{1-x^2}dx = \coth^{-1}x + C, \quad |x| > 1$$

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Formulae

DERIVATIVES

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{d}{dx}[u^n] = nu^{n-1}\frac{du}{dx}$$

$$\frac{d}{dx}\{[f(g(x))]\} = f'[(g(x)]g'(x)$$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dv}\cdot\frac{dv}{dx}$$

QUADRATIC FORMULA

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$