CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2011/12

COURSE NAME	:	GEOTECHNICS
COURSE CODE	:	BFC3033/ BFC31703
PROGRAMME	:	3 BFF
DATE	:	JANUARY 2012
DURATION	:	3 HOURS
INSTRUCTIONS	:	ANSWER ANY **FOUR (4)** QUESTIONS ONLY

THIS PAPER CONSISTS OF **NINETEEN (19)** PAGES

CONFIDENTIAL

Q1 (a) The moist mass of a soil specimen is 23.3 kg. The specimen's dried mass is 17.1 kg and the volume measured before drying is 0.015 m^3. If the specific gravity (G_s) of the soil is 2.65, determine the following;

(i) Void ratio.
(i) Degree of saturation.
(ii) Percentage of air
(iii) Dry unit weight.

(8 marks)

(b) In order to classify a clayey soil obtained from a construction site, there are a few laboratory tests that have to be conducted on the soil sample. Describe briefly two (2) of the tests.

(4 marks)

(c) Figure **Q1(c)** gives the grain size distribution of two soils obtained from sieve and hydrometer analysis. The liquid limit and plastic limit of No. 40 sieve fraction of the soils are as shown in **Table 1**;

Table 1

	Soil A	**Soil B**
Liquid limit	32	27
Plastic limit	22	21

(i) Classify both the soils according to the Unified Soil Classification System (USCS).

(6 marks)

(ii) Classify soil B according to the American Association of State Highways and Transportation Officials (AASHTO) System.

(3 marks)

(iii) Comment the suitability of Soil B to be used as a sugrade for the construction of a road.

(4 marks)

Q2 (a) Derive the equation for the evaluation of critical hydraulic gradient in a soil of specific gravity, G_s and void ratio, e.

(5 marks)

(b) It is practically impossible to collect an undisturbed sample of dry clean sand to determine its coefficient of permeability in the laboratory. Suggest an alternative method of determining the permeability coefficient of a surface sand deposit to obtain an appropriate value of it for design purposes.

(5 marks)

(c) Figure **Q2(c)** shows the flow net and the necessary dimensions for the flow of water in an isotropic soil (permeability coefficient is 20 x 10^{-3} cm/sec) beneath a concrete dam. The dam is embedded 1 m into the ground surface. The figure further shows a 5 m deep sheet pile cut off wall located at the heel of the dam. The upstream head water is 7 m deep but the tail water is at the ground surface.

(i) Determine the rate of discharge of water to the downstream per meter width of the dam

(3 marks)

(ii) Determine the uplift water pressure on the base of the dam at the heel and at the toe of the dam.

(4 marks)

(iii) Determine the pore water pressure at the point P on the impermeable stratum boundary?

(3 marks)

(iv) If the properties of the soil are e = 0.6 and G_s = 2.70, analyse the factor of safety against piping.

(5 marks)

Q3 (a) There are two common types of retaining walls in the field, which are rigid retaining walls and mechanically stabilized earth walls. List and sketch **TWO (2)** for each type.

(5 marks)

(b) The stresses acting on a soil element are as shown in Figure **Q3 (b)**. Determine the magnitude and the orientation of:

(i) the maximum and the minimum principal stresses

(ii) the normal and the shear stresses on the plane EF

(10 marks)

(c) A 6-m high retaining wall is as shown in Figure **Q3 (c)**.

(i) Plot the variation of the total pressure, the pore water pressure and the effective pressure with depth for the entire soil profile

(ii) Determine Rankine active force per unit length of the wall and the location of the resultant

(10 marks)

Q4 (a) The consolidation process of a fully saturated clayey soil can best be explained by a model known as piston-spring analogy. Briefly, explains the process by using the piston spring analogy.

(5 marks)

(b) The results of a laboratory consolidation test on undisturbed clayey soil sample are as shown in **Table 2**.

Table 2

Pressure, σ' (kN/m^2)	**Voids ratio, e**
25	1.030
50	1.020
100	0.980
200	0.910
400	0.790
800	0.710
1600	0.620
800	0.635
400	0.655
20	0.670

(i) Plot an e-log σ', and

(ii) Determine the preconsolidation pressure, p_c, the compression index, C_c, and the swelling index, C_s.

(10 marks)

(c) If the sample in **Q4(b)** above was obtained from a normally consolidated clay layer of 6m thickness with an initial void ratio of 1.10, and experiencing a uniform increase in stress of 80 kN/m^2, calculate the primary consolidation settlement of the layer?

(5 marks)

(d) Discuss the changes in the primary consolidation settlement if the clay layer as in **Q4(c)** is overconsolidated and having an overconsolidation ratio of 1.5.

(5 marks)

Q5 (a) Briefly, describe and discuss the advantages of Vane Shear Test, Direct Shear Test and Triaxial Test.

(6 marks)

(b) Outline the shear strength parameters that you would apply in design and checking the stability of a structure founded on soft clay immediately after construction and after a long term?

(4 marks)

(c) A consolidated undrained (CU) triaxial test with pore pressure measurements was performed on a soil sample. The observations at the point of failure are given in **Table 3**:

Table 3

	Test 1	**Test 2**	**Test 3**
σ_3 (kPa)	40	60	90
σ_1 (kPa)	84	132	194
u (kPa)	22	34	40

(i) Plot the total and effective stress failure envelope on a Mohr-Coulomb diagram and hence determine the shear strength parameters in terms of total and effective stress.

(10 marks)

(ii) If a fourth sample was consolidated to 50 kN/m^2, calculate the axial stress and pore water pressure at failure

(5 marks)

Q6 (a) List out **FOUR (4)** methods to stabilize the failure slopes.

(5 marks)

(b) Most slope failures in Malaysia occur during rainy seasons. Discuss why do slope failures usually occur during these seasons.

(5 marks)

(c) A homogenous slope will be constructed at a new development area for community area. Assumed there is reduction on a pore water pressure due to the construction stages and pore water pressure may be predicted via r_u which its average value is 0.40. From the laboratory works, undrained cohesion and undrained internal friction angle are 0 kPa and 35°, respectively.

The slope was divided into **seven (7)** slices and all the required data for the slices are as shown in **Table 4**.

Table 4

Slice No.	**W (kN/m)**	**α (°)**	**b_n (m)**
1	147	-13	1.5
2	477	-4.5	2
3	917	6	2
4	1192	17	2
5	1338	28	2
6	1284	40.5	2
7	825	53.5	2.35

Determine the factor of safety against sliding for the trial slip surface for this proposed slope. Use the ordinary method of slices with the following equation:

$$\textbf{FS} = \frac{\sum c_n L_n + (W_n \cos \alpha_n - r_u \sec \alpha_n) \tan \phi_n}{\sum W_n \sin \alpha_n}$$

(15 marks)

S1 (a) Jisim tanah basah untuk satu spesimen tanah 23.3 kg. Jisim kering sepsimen berkenaan ialah 17.1kg dan isipadu sabelum ianya dikeringkan ialah 0.015 m^3. Jika graviti tentu (G_s) tanah ialah 2.65, tentukan parameter berikut;

(i) Nisbah lompang.
(ii) Darjah ketepuan.
(iii) Peratus udara.
(iv) Berat unit kering.

(8 markah)

(b) Untuk mengkelaskan suatu tanah liat yang diperolehi daripada tapak bina, beberapa ujikaji-ujikaji makmal hendaklah dilakukan ke atas sampel tanah yang diambil daripada tapak bina. Terangkan dengan ringkas dua daripada ujikaji-ujikaji berkenaan.

(4 markah)

(c) Rajah **Q1(c)** menunjukkan taburan saiz zarah untuk dua jenis tanah yang didapati dari analisis ayakkan dan meter hidro. Had cecair dan plastik masing-masing untuk bahagian yang lulus ayak no. 40 bagi kedua-dua tanah adalah seperti di **Jadual 1**:

Jadual 1

	Tanah A	**Tanah B**
Had cecair	32	27
Had plastik	22	21

(i) Kelaskan kedu-dua tanah mengikut sistem pengkelasan Sistem Pengkelasan Tanah Bersekutu (USCS).

(6 markah)

(ii) Kelaskan Tanah B mengikut Sistem Pertubuhan Lebuhraya Persekutuan dan Pegawai-pegawai Pengangkutan Amerika (AASHTO).

(3 markah)

(iii) Komen tentang kesesuain Tanah B untuk digunakan sebagai sub-gred bagi pembinaan sebatang jalan.

(4 markah)

S2 (a) Terbitkan persamaan untuk menentukan kecerunan hidraul untuk sesuatu tanah yang graviti tentu, G_s dan nisbah lompang, e.

(5 markah)

(b) Adalah amat mustahil untuk mendapatkan sampel tak terusik bagi pasir bersih bagi menentukan pekali ketelapan tanah berkenaan di makmal. Cadangkan suatu kaedah lain untuk menentukan nilai yang munasabah pekali ketelapan tanah sebegini bagi tujuan rekabentuk.

(5 markah)

(c) Figure **Q2(c)** menunjukkan jaringan aliran dan dimensi-dimensi yang diperlukan untuk aliran air dibawah empangan konkrit bagi tanah isotropik (pekali ketelapan tanah ialah 20 x 10^{-3} cm/sec) . Empangan berkenaan dibina pada kedalaman 1 m daripada permukaan tanah. Rajah berkenaan juga menunjukkan satu cerucuk keping sedalam 5 m yang dibina sebelah di hadapan empangan. Paras air di hulu ialah sedalam 7 m manakala paras air di hilir ialah pada aras permukaan bumi.

(i) Tentukan kadaralir keluar air dihulu untuk satu meter lebar empangan.

(3 markah)

(ii) Tentukan tekanan tujahan air pada tapak dihadapan dan ditumit empangan.

(4 markah)

(iii) Kirakan tekanan air liang pada titik P diatas permukaan tak telap seperti yang ditunjukkan dalam rajah.

(3 markah)

(iv) Jika ciri-ciri tanah berkenaan ialah e = 0.6 dan G_s = 2.70, analisiskan faktor keselamatan terhadap pempaipan.

(5 markah)

S3 (a) Terdapat dua (2) jenis tembok penahan yang umum di lapangan, iaitu tembok penahan tegar dan tembok terstabil tanah secara mekanikal. Senarai dan lukiskan **dua (2)** daripada setiap jenis tersebut.

(5 markah)

(b) Tegasan-tegasan yang bertindak keatas satu elemen tanah adalah seperti yang ditunjukkan dalam Rajah **Q3 (b)**. Tentukan magnitud dan penghalaan untuk:

(i) tegasan-tegasan utama maksimum dan minimum.

(ii) tegasan-tegasan normal dan riceh diatas satah

(10 markah)

(c) Suatu tembok penahan yang tingginya 6-m adalah seperti yang ditunjukkan dalam Rajah **Q3 (c)**.

(i) Plot perubahan tekanan jumlah, tekanan air liang dan tekanan kesan berbanding kedalaman keseluruhan profail tanah berkenaan.

(ii) Tentukan daya aktif per meter panjang tembok dan kedudukan daya paduan tersebut.

(10 markah)

S4 (a) Proses pengukuhan untuk tanah liat tepu sepenuhnya boleh diterangkan dengan baik menggunakan model yang dikenali sebagai analogi spring dan omboh. Terangkan dengan ringkas proses berkenaan dengan menggunakan analogi spring dan omboh.

(5 markah)

(b) Keputusan ujikaji pengukuhan yang dijalankan dalam makmal keatas satu sampel tanah liat adalah seperti yang ditunjukkan dalam **Jadual 2**.

Jadual 2

Tekanan, σ' (kN/m^2)	**Nisbah lompang, e**
25	1.030
50	1.020
100	0.980
200	0.910
400	0.790
800	0.710
1600	0.620
800	0.635
400	0.655
20	0.670

(i) Plot lengkung e-log σ', dan

(ii) Tentukan tegasan pra pengukuhan, p_c, indeks mampatan, C_c, dan indeks pengembangan, C_s

(10 markah)

(c) Jika sampel seperti di **S4(b)** di atas diambil daripada lapisan tanah liat terkukuh biasa yang ketebalannya ialah 6 m dan mempunyai nisbah lompang awal bernilai 1.10, serta mengalami perubahan tegasan bernilai 80 kN/m^2, kirakan pengenapan pengukuhan primer lapisan ini?

(5 markah)

(d) Bincangkan perubahan-perubahan yang harus dilakukan dalam menentukan pengenapan pengukuhan primer jika lapisan tanah liat seperti di **S4(c)** ialah lapisan terkukuh lebih yang mempunyai nisbah terkukuh lebih bernilai 1.5.

(5 markah)

S5 (a) Terangkan dan bincang kelebihan-kelebihan ujian ram riceh, ujian ricih terus dan ujian tiga paksi.

(6 markah)

(b) Gariskan parameter kekuatan ricih yang anda akan gunakan dalam tujuan rekabentuk dan semakan kestabilan struktur yang dibina diatas tanah liat lembut sebaik sahaja tamat pembinaan dan selepas jangkamasa yang panjang?

(4 markah)

(c) Satu ujikaji tiga paksi terkukuh tak tersalir (CU) dengan mengukur tekanan liang telah dijalankan keatas satu sampel tanah. Data cerapan yang dilakukan semasa kegagalan ialah seperti di **Jadual 3**.

Jadual 3

	Ujikaji 1	**Ujikaji 2**	**Ujikaji 3**
σ_3 (kPa)	40	60	90
σ_1 (kPa)	84	132	194
u (kPa)	22	34	40

(i) Plot linkungan kegagalan untuk tegasan-tegasan jumlah dan kesan mengunakan rajah Mohr-Coulomb dan tentukan parameter-parameter berdasarkan tegasan jumlah dan tegasan kesan.

(10 markah)

(ii) Jika sampel keempat dikukuhkan sehingga 50 kN/m^2, kirakan tegasan sisih dan tekanan air liang apabila kegagalan berlaku.

(5 markah)

S6 (a) Senaraikan **EMPAT (4)** kaedah menstabilkan cerun yang gagal.

(5 markah)

(b) Kebanyakkan kegagalan cerun di Malaysia berlaku semasa musim tengkujuh. Bincangkan kenapa kegagalan cerun kerap berlaku pada musim-musim sabegini.

(5 markah)

(c) Satu cerun homogenus akan dibina pada satu kawasan komuniti yang baru. Anggapkan terdapat pengurangan pada tekanan air liang yang disebabkan oleh peringkat pembinaan dan tekanan air liang boleh diramalkan melalui r_u yang mana nilai puratanya ialah 0.40. Daripada kerja-kerja makmal, kejeleketan tak tersalir dan sudut geseran dalaman masing-masing ialah 0 kPa dan 35°.

Cerun berkenaan dibahagikan kepada **tujuh (7)** hirisan dan data-data yang diperlukan bagi setiap hirisan adalah seperti di **Jadual 4.**

Jadual 4

No Hirisan	W (kN/m)	α (°)	b_n (m)
1	147	-13	1.5
2	477	-4.5	2
3	917	6	2
4	1192	17	2
5	1338	28	2
6	1284	40.5	2
7	825	53.5	2.35

Untuk permukaan gelinciran cubaan bulat seperti yang diberikan itu, anggarkan faktor keselamatan, FK.

$$\mathbf{FK} = \frac{\sum c_n L_n + (W_n \, kos\, \alpha_n - r_u sek\, \alpha_n)\tan\phi_n}{\sum W_n \sin\alpha_n}$$

(15 markah)

FINAL EXAMAMINATION

SEMESTER/SESSION : I/2011/2012		PROGRAMME : 3 BFC	
COURSE NAME : GEOTECHNICS		COURSE CODE : BFC 3033/BFC31703	

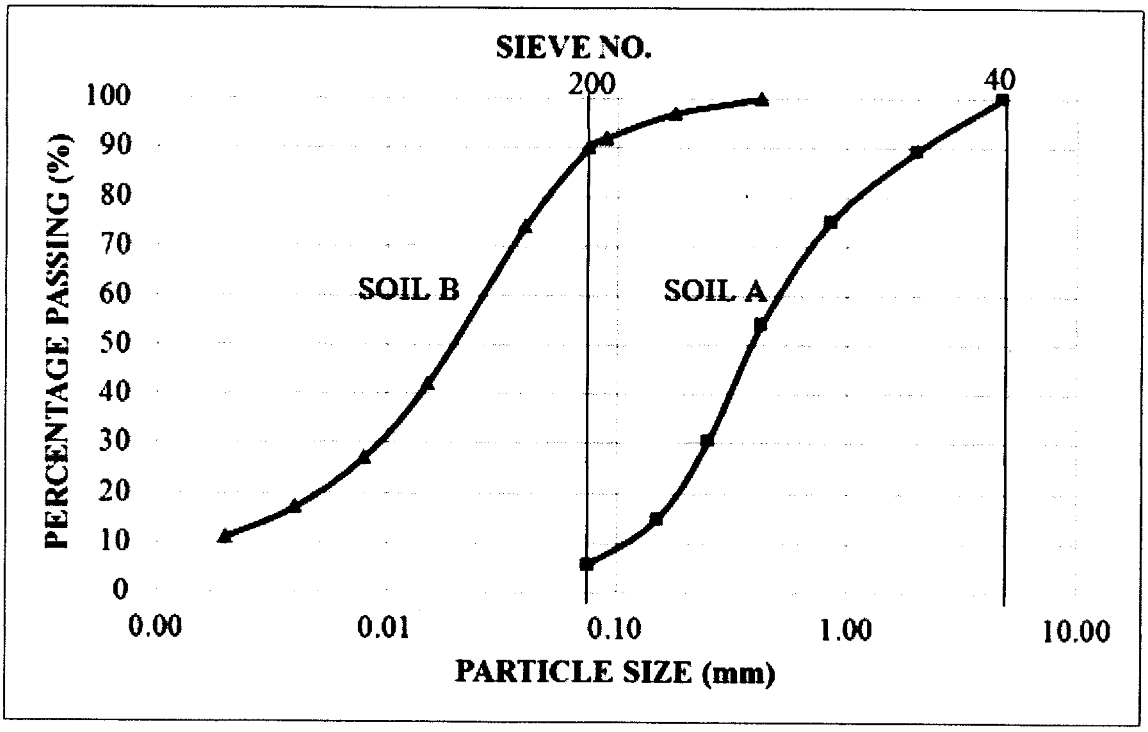


Figure Q1(c)

FINAL EXAMAMINATION

SEMESTER/SESSION : I/2011/2012 PROGRAMME : 3 BFC
COURSE NAME : GEOTECHNICS COURSE CODE : BFC 3033/BFC31703

General Classification	**Granular Materials (35% or less passing the 0.075 mm sieve)**						
Group Classification	A-1		A-3	A-2			
	A-1-a	A-1-b		A-2-4	A-2-5	A-2-6	A-2-7
Sieve Analysis, % passing							
2.00 mm (No. 10)	50 max	...	...	...	...	...	...
0.425 (No. 40)	30 max	50 max	51 min	...	...	...	...
0.075 (No. 200)	15 max	25 max	10 max	35 max	35 max	35 max	35 max
Characteristics of fraction passing 0.425 mm (No. 40)							
Liquid Limit	...		...	40 max	41 min	40 max	41 min
Plasticity Index	6 max		N.P.	10 max	10 max	11 min	11 min
Usual types of significant constituent materials	stone fragments, gravel and sand		fine sand	silty or clayey gravel and sand			
General rating as a subgrade	excellent to good						

<u>Note (1):</u> Plasticity index of A-7-5 subgroup is equal to or less than the LL - 30. Plasticity index of A-7-6 subgroup is greater than LL - 30

Figure Q1(c)(ii): AASHTO Classification System

FINAL EXAMAMINATION

SEMESTER/SESSION : I/2011/2012 PROGRAMME : 3 BFC
COURSE NAME : GEOTECHNICS COURSE CODE : BFC 3033/BFC31703

General Classification	**Silt-Clay Materials (>35% passing the 0.075 mm sieve)**			
Group Classification	A-4	A-5	A-6	A-7 A-7-5 A-7-6
Sieve Analysis, % passing				
2.00 mm (No. 10)	...	...	...	...
0.425 (No. 40)	...	...	...	...
0.075 (No. 200)	36 min	36 min	36 min	36 min
Characteristics of fraction passing 0.425 mm (No. 40)				
Liquid Limit	40 max	41 min	40 max	41 min
Plasticity Index	10 max	10 max	11 min	11 min[1]
Usual types of significant constituent materials	silty soils		clayey soils	
General rating as a subgrade	fair to poor			

Note (1): Plasticity index of A-7-5 subgroup is equal to or less than the LL - 30. Plasticity index of A-7-6 subgroup is greater than LL - 30

Figure Q1(c)(ii): AASHTO Classification System

FINAL EXAMAMINATION

SEMESTER/SESSION	: I/2011/2012	PROGRAMME	: 3 BFC
COURSE NAME	: GEOTECHNICS	COURSE CODE	: BFC 3033/BFC31703

UNIFIED SOIL CLASSIFICATION AND SYMBOL CHART

Major divisions	Group symbol	Typical names
COARSE-GRAINED SOILS (more than 50% of material is larger than No. 200 sieve size)		
		Clean Gravels (Less than 5% fines)
GRAVELS More than 50% of coarse fraction larger than No. 4 sieve size	GW	Well-graded gravels, gravel-sand mixtures, little or no fines
	GP	Poorly-graded gravels, gravel-sand mixtures, little or no fines
		Gravels with fines (More than 12% fines)
	GM	Silty gravels, gravel-sand-silt mixtures
	GC	Clayey gravels, gravel-sand-clay mixtures
		Clean Sands (Less than 5% fines)
SANDS 50% or more of coarse fraction smaller than No. 4 sieve size	SW	Well-graded sands, gravelly sands, little or no fines
	SP	Poorly graded sands, gravelly sands, little or no fines
		Sands with fines (More than 12% fines)
	SM	Silty sands, sand-silt mixtures
	SC	Clayey sands, sand-clay mixtures
FINE-GRAINED SOILS (50% or more of material is smaller than No. 200 sieve size)		
SILTS AND CLAYS Liquid limit less than 50%	ML	Inorganic silts and very fine sands, rock flour, silty of clayey fine sands or clayey silts with slight plasticity
	CL	Inorganic clays of low to medium plasticity, gravelly clays, sandy clays, silty clays, lean clays
	OL	Organic silts and organic silty clays of low plasticity
SILTS AND CLAYS Liquid limit 50% or greater	MH	Inorganic silts, micaceous or diatomaceous fine sandy or silty soils, elastic silts
	CH	Inorganic clays of high plasticity, fat clays
	OH	Organic clays of medium to high plasticity, organic silts
HIGHLY ORGANIC SOILS	PT	Peat and other highly organic soils

LABORATORY CLASSIFICATION CRITERIA

GW	$C_u = \frac{D_{60}}{D_{10}}$ greater than 4; $C_c = \frac{D_{30}}{D_{10} \times D_{60}}$ between 1 and 3	
GP	Not meeting all gradation requirements for GW	
GM	Atterberg limits below "A" line or P.I. less than 4	Above "A" line with P.I. between 4 and 7 are borderline cases requiring use of dual symbols
GC	Atterberg limits above "A" line with P.I. greater than 7	
SW	$C_u = \frac{D_{60}}{D_{10}}$ greater than 4; $C_c = \frac{D_{30}}{D_{10} \times D_{60}}$ between 1 and 3	
SP	Not meeting all gradation requirements for GW	
SM	Atterberg limits below "A" line or P.I. less than 4	Limits plotting in shaded zone with PI between 4 and 7 are borderline cases requiring use of dual symbols
SC	Atterberg limits above "A" line with P.I. greater than 7	

Determine percentages of sand and gravel from grain-size curve. Depending on percentage of fines (fraction smaller than No. 200 sieve size), coarse-grained soils are classified as follows:

Less than 5 percent .... GW, GP, SW, SP
More than 12 percent .... GM, GC, SM, SC
5 to 12 percent .... Borderline cases requiring dual symbols

PLASTICITY CHART

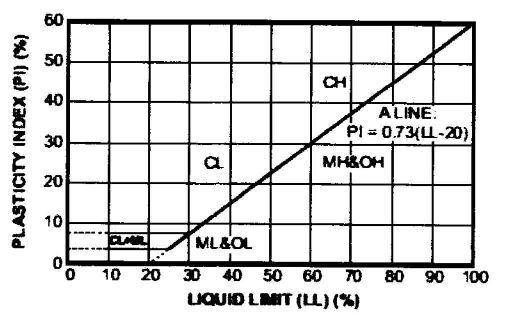
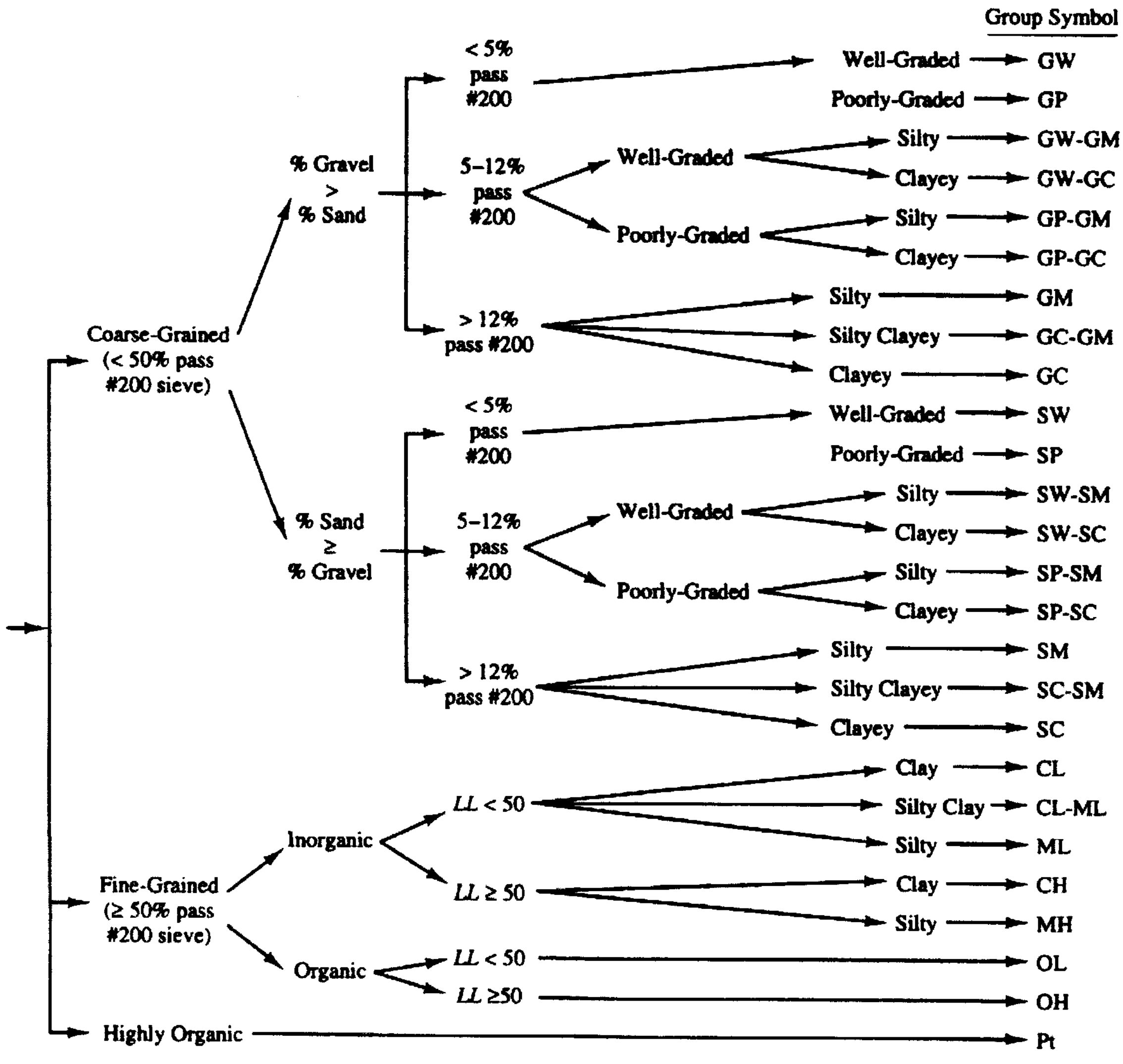


Figure Q1(c)(i): USCS Classification System

FINAL EXAMAMINATION

SEMESTER/SESSION : I/2011/2012 PROGRAMME : 3 BFC
COURSE NAME : GEOTECHNICS COURSE CODE : BFC 3033/BFC31703



<u>Figure Q1(c)(i)</u>: USCS Classification System

FINAL EXAMAMINATION

SEMESTER/SESSION : I/2011/2012 PROGRAMME : 3 BFC
COURSE NAME : GEOTECHNICS COURSE CODE : BFC 3033/BFC31703

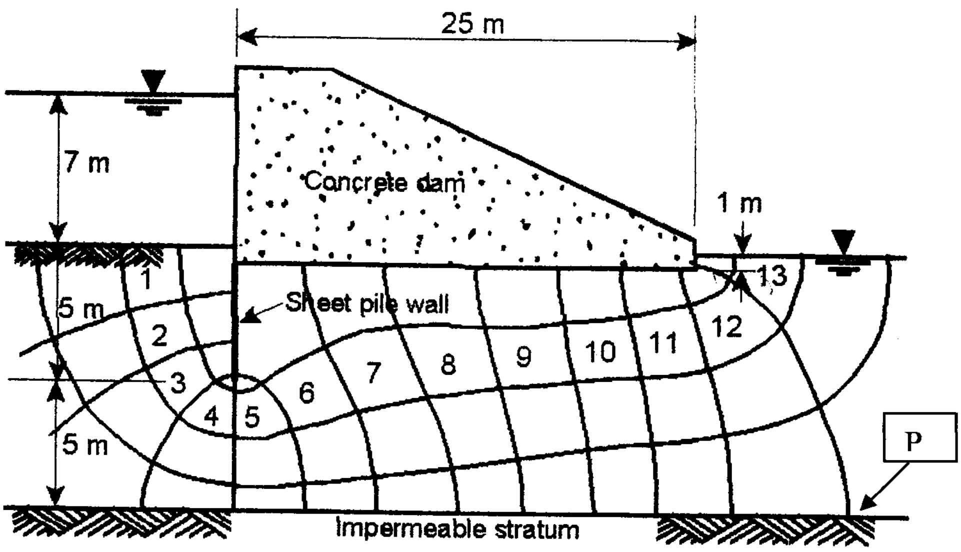
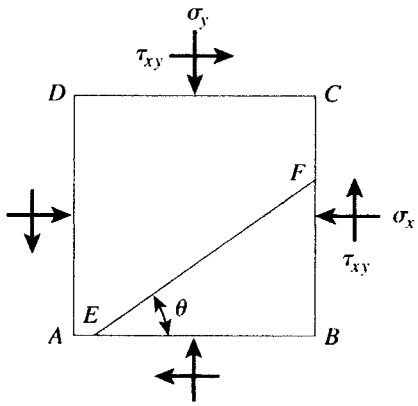


Figure Q2(c)

FINAL EXAMAMINATION

SEMESTER/SESSION : I/2011/2012 PROGRAMME : 3 BFC
COURSE NAME : GEOTECHNICS COURSE CODE : BFC 3033/BFC31703



Where,

$\sigma_y = 200$ kPa
$\sigma_x = 100$ kPa
$\tau_{xy} = 50$ kPa
$\theta = 30°$

(a)

Figure Q3(b):

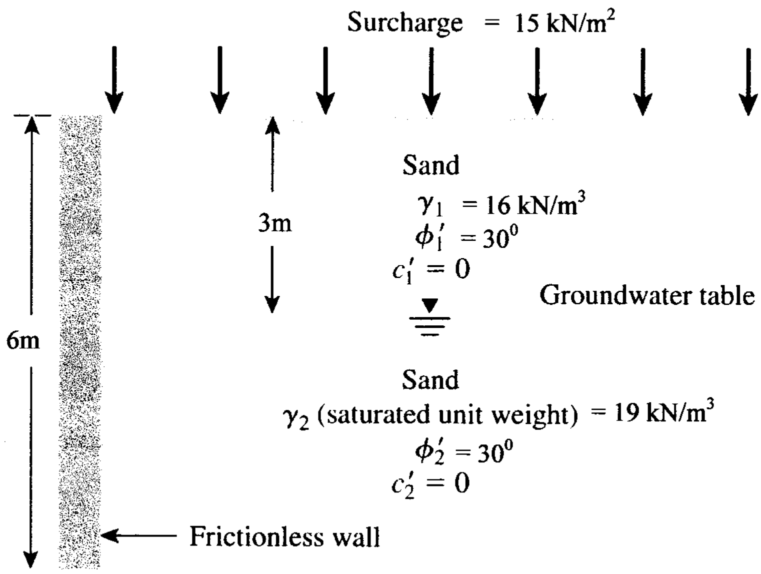


Figure Q3(c):

FINAL EXAMAMINATION

SEMESTER/SESSION	: I/2011/2012	PROGRAMME	: 3 BFC
COURSE NAME	: GEOTECHNICS	COURSE CODE	: BFC 3033/BFC31703

BASIC GEOTECHNICS EQUATION SHEET

$$\gamma_d = \frac{G\gamma_w}{1+e} \qquad \gamma_B = \frac{(G+Se)\gamma_w}{1+e} \qquad w = \frac{Se}{G}$$

$$k = \frac{QL}{Aht} \;; \qquad k = 2.303\frac{aL}{At}\log_{10}\frac{h_1}{h_2}; \qquad k = \frac{2.303q\log_{10}\left(\frac{r_2}{r_1}\right)}{\pi\left(h_1^2 - h_2^2\right)} \;; \qquad k = \frac{q\log_{10}\left(\frac{r_1}{r_2}\right)}{2.727H(h_1 - h_2)}$$

$$k_H = \frac{1}{H}(k_{H_1}H_1 + k_{H_2}H_2 + \dots\dots\dots + k_{H_n}H_n) \;; \qquad k_V = \frac{H}{\left(\frac{H_1}{k_{V_1}}\right) + \left(\frac{H_2}{k_{V_2}}\right) + \dots\dots + \left(\frac{H_n}{k_{V_n}}\right)}$$

$$q = k\frac{HN_f}{N_d}$$

$$\sigma_n = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y + \sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$Sc = \frac{C_s H}{1+e_0}\log\left(\frac{\sigma_o' + \Delta\sigma'}{\sigma_c'}\right) \;; \qquad Sc = \frac{C_S H}{1+e_0}\log\frac{\sigma_c'}{\sigma_0'} + \frac{C_c H}{1+e_0}\log\left(\frac{\sigma_0' + \Delta\sigma'}{\sigma_c'}\right)$$

$$T_v = \frac{c_v t}{H_{dr}^2}$$

$$\tau_f = c + \sigma\tan\phi \;; \qquad \sigma_1' = \sigma_3'\tan^2\left(45 + \frac{\phi'}{2}\right) + 2c'\tan\left(45 + \frac{\phi'}{2}\right)$$

$$K_0 = (1 - \sin\phi')$$

$$K_a = \frac{\sigma_a'}{\sigma_0'} = \tan^2\left(45 - \frac{\phi'}{2}\right) = \frac{1-\sin\phi'}{1+\sin\phi'} \;; \qquad K_p = \frac{\sigma_p'}{\sigma_0'} = \tan^2\left(45 + \frac{\phi'}{2}\right) = \frac{1+\sin\phi'}{1-\sin\phi'}$$

$$z_0 = \frac{2c'}{\gamma\sqrt{K_a}}; \qquad z_0 = \frac{2c_u}{\gamma};$$