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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	PARTIAL DIFFERENTIAL EQUATION
COURSE CODE	:	BWA 30303
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

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Q1 (a) If $z = x\,f(x+y) + y\,g(x+y)$ where f and g are arbitrary functions, form a second order partial differential equation

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = k\frac{\partial^2 z}{\partial x \partial y},$$

where k is a constant to be determined.

(3 marks)

(b) Solve the following traffic flow problem,

$$\frac{\partial u}{\partial t} + (1-2u)\frac{\partial u}{\partial x} = 0, \quad u(x,\, 0) = x\left(x^2+1\right)^{-1}.$$

(5 marks)

(c) A periodic function $f(x)$ is defined by

$$f(x) = \begin{cases} -\cos(x), & -\pi < x \le 0, \\ \cos(x), & 0 < x < \pi, \end{cases}$$

and $f(x) = f(x+2\pi)$.

i) Show that the Fourier series is given by

$$f(x) = \frac{8}{\pi}\sum_{n=1}^{\infty}\frac{n\sin(2nx)}{(2n-1)(2n+1)}.$$

(7 marks)

ii) By choosing an appropriate value of x, deduce the sum of the infinite series

$$\frac{1}{1\cdot 3} - \frac{3}{5\cdot 7} + \frac{5}{9\cdot 11} - \dots\ .$$

(4 marks)

Q2 Given

$$\frac{\partial^2 u}{\partial t^2}+2(t-1)\frac{\partial^2 u}{\partial x\partial t}-4t\frac{\partial^2 u}{\partial x^2}+\frac{2}{1+t}\frac{\partial u}{\partial x}-\frac{1}{1+t}\frac{\partial u}{\partial t}=0,$$

for $-\infty < x < \infty,\ t > 0$.

a) Show that the above equation is hyperbolic type and

$$\zeta = x - t^2 \text{ and } \eta = x + 2t,$$

are characteristic coordinates for the equation.

(8 marks)

(b) Obtain the canonical form of the equation and find the general solution.

(5 marks)

(c) If the initial conditions are

$$u(x,\ 0) = f(x) \text{ and } \frac{\partial u}{\partial t}(x,\ 0) = g(x), \text{ for } -\infty < x < \infty,$$

show that the general solution can be written as

$$u(x,\ t) = f\left(x - t^2\right) + \frac{1}{2}\int_{x-t^2}^{x+2t} g(\tau)\, d\tau .$$

(9 marks)

Q3 (a) Find a formal solution of a vibrating string with fixed ends:

$$\begin{aligned} &u_{tt} - c^2 u_{xx} = 0, && 0 < x < L,\ t > 0 \\ &u(0,t) = u(L,t) = 0, && t \geq 0 \\ &u(x,0) = f(x),\ u_t(x,0) = g(x), && 0 \leq x \leq L \end{aligned}$$

using the separation of variables method.

(15 marks)

(b) Prove that the solution in **Q3 (a)** can be represented as a superposition of a forward and a backward wave.

(4 marks)

Q4 The voltage distribution in an electric transmission line is given by

$$v_t = kv_{xx} \text{ for } 0 < x < l,\ t > 0.$$

A voltage equal to zero is maintained at $x = l$, while at the end $x = 0$, the voltage varies according to the

$$v(0,\ t) = Ct,\ t > 0,$$

where C is a constant. Find $v(x,t)$ if the initial voltage distribution is zero.

(20 marks)

Q5 Consider the Laplacian in cylindrical polar coordinates

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}=0,$$

in the disk $a \le r \le b$ and $0 < \theta < 2\pi$. Use the method of separation of variables to derive the general solution for u in this region. Next, given the boundary conditions

$$u(a,\ \theta) = T_1,\ u(b,\ \theta) = T_2,$$

write down the exact solution.

(20 marks)

- END OF QUESTIONS -

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Formulae

Fourier Series: $f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}\left\{a_n\cos\left(\frac{n\pi x}{\ell}\right)+b_n\sin\left(\frac{n\pi x}{\ell}\right)\right\}$,

where $a_0=\frac{1}{\ell}\int_{-\ell}^{\ell}f(x)dx$,

$$a_n=\frac{1}{\ell}\int_{-\ell}^{\ell}f(x)\cos\left(\frac{n\pi x}{\ell}\right)dx,\quad n=1,2,3,\ldots,$$

$$b_n=\frac{1}{\ell}\int_{-\ell}^{\ell}f(x)\sin\left(\frac{n\pi x}{\ell}\right)dx,\quad n=1,2,3,\ldots$$

Half Range Cosine Series: $f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}a_n\cos\left(\frac{n\pi x}{\ell}\right)$,

where $a_0=\frac{2}{\ell}\int_{0}^{\ell}f(x)dx$,

$$a_n=\frac{2}{\ell}\int_{0}^{\ell}f(x)\cos\left(\frac{n\pi x}{\ell}\right)dx,\quad n=1,2,3,\ldots$$

Half Range Sine Series: $f(x)=\sum_{n=1}^{\infty}b_n\sin\left(\frac{n\pi x}{\ell}\right)$,

where $b_n=\frac{2}{\ell}\int_{0}^{\ell}f(x)\sin\left(\frac{n\pi x}{\ell}\right)dx,\quad n=1,2,3,\ldots$