Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2017/2018

COURSE NAME	:	NUMERICAL ANALYSIS I
COURSE CODE	:	BWA 20903
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JUNE / JULY 2018
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS B) ALL CALCULATIONS MUST BE IN **4 DECIMAL** PLACES

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) Show that the Midpoint method and the Modified Euler method give the same approximations to the initial-value problem

$$y' = -y + x + 1, \qquad 0 \le x \le 1, \qquad y(0) = 1,$$

for any choice of h. Why is this true?

(10 marks)

(b) Solve the following initial value problem by using Adams-Bashforth Three-Step Explicit method. Use exact values to find $y(0.2)$ and $y(0.4)$.

$$y' = 1 + \frac{y}{x}, \qquad 1 \le x \le 2, \quad y(1) = 2, \quad \text{with} \quad h = 0.2.$$

Compare the results to the actual values by finding the absolute errors if the actual solution is

$$y(x) = x \ln x + 2x.$$

(10 marks)

(c) The following methods can be used to solve an ODE $x' = f(t, x)$.

(i) $x_{n+1} = x_n + \frac{1}{2}\left[f(t_n, x_n) + f(t_{n+1}, x_n + f(t_n, x_n)h) \right] h$.

(ii) $x_{n+1} = x_n + \frac{1}{2}\left[3f(t_n, x_n) - f(t_{n-1}, x_{n-1}) \right] h$.

(iii) $x_{n+1} = x_n + \frac{1}{2}\left[f(t_n, x_n) + f(t_{n+1}, x_{n+1}) \right] h$.

For each method, determine if it has the following properties: single-step method, implicit method and Runge-Kutta type method.

(5 marks)

Q2 (a) Given a symmetric matrix

$$A = \begin{pmatrix} 5 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 5 \end{pmatrix}.$$

(i) Find the dominant eigenvalue and corresponding eigenvector for matrix A by using power method with $v^{(0)} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}^T$ and $\varepsilon = 0.005$.

(ii) Find the smallest eigenvalue (in absolute value) and corresponding eigenvector for matrix A by using shifted power method with $v^{(0)} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}^T$ and $\varepsilon = 0.005$.

(10 marks)

(b) Find the interval of which the eigenvalues of matrix A below are contained by using Gerschgorin's theorem.

$$A=\begin{pmatrix} 5 & 0 & 0 & 1 \\ 1 & 0 & -1 & 1 \\ -1.5 & 1 & -2 & 1 \\ -1 & 1 & 3 & -3 \end{pmatrix}.$$

(9 marks)

(c) Find the integral between $x=1.0$ and 1.8 for the data in **Table Q2(c)** using Simpson's 1/3 rule with $h=0.4$ and then with $h=0.2$. From these two results, extrapolate to get a better result using Romberg interpolation method.

Table Q2(c)

x	1.0	1.2	1.4	1.6	1.8
$f(x)$	1.54	1.81	2.15	2.58	3.11

(6 marks)

Q3 (a) The following four equations are proposed to compute $21^{1/3}$. Rank them in order, based on their apparent speed of convergence, assuming $x_0=1$.

$$g_1(x)=x-\frac{x^3-21}{3x^2},\qquad g_2(x)=x-\frac{x^4-21x}{x^2-21},\qquad g_3(x)=\left(\frac{21}{x}\right)^{1/2}.$$

(8 marks)

(b) Given a system of linear equations as below:

$$\begin{aligned} 2x_1 + 4x_2 - x_3 &= 1 \\ 4x_1 + x_2 + x_3 &= -2 \\ 2x_1 - 3x_2 + 6x_3 &= 1 \end{aligned}$$

(i) Write down the equations in matrix form $Ax=B$.

(ii) Solve the system by using Doolittle method.

(iii) Given that the inverse of A is

$$A^{-1}=\frac{1}{56}\begin{pmatrix} -9 & 21 & -5 \\ 22 & -14 & 6 \\ 14 & -14 & 14 \end{pmatrix}.$$

Determine and analyse the condition number of A.

(17 marks)

Q4 (a) Let $A = \begin{bmatrix} \alpha & 1 & 0 \\ \beta & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$.

Find the conditions or values of α and β for which

(i) A is singular.

(ii) A is strictly diagonally dominant.

(iii) A is symmetric.

(iv) A is positive definite.

(8 marks)

(b) Consider the data (0, 0), (0.5, α), (1, 3) and (2, 2). Use Lagrange polynomial interpolation to find the value of α if the coefficient of x^3 in the polynomial is 6.

(13 marks)

(c) State the weakness of using Lagrange polynomial interpolation and the advantages of using Newton's divided difference when constructing a polynomial.

(4 marks)

- END OF QUESTIONS -

FINAL EXAMINATION

SEMESTER/SESSION: SEM II / 2017/2018 PROGRAMME CODE: BWA
COURSE NAME : NUMERICAL ANALYSIS I COURSE CODE : BWA 20903

Midpoint method:

$$y_{i+1} = y_i + k_2,\ i = 0,1,2,\ldots$$

where

$$k_1 = hf(x_i,\ y_i)$$

$$k_2 = hf(x_i + \frac{h}{2},\ y_i + \frac{k_1}{2})$$

Modified Euler method:

$$y_{i+1} = y_i + \frac{1}{2}k_1 + \frac{1}{2}k_2$$

where

$$k_1 = hf(x_i,\ y_i)$$

$$k_2 = hf(x_i + h,\ y_i + k_1)$$

Adams-Bashforth Three-Step Explicit method:

$$y_{i+1} = y_i + \frac{h}{12}\left[23f(x_i, y_i) - 16f(x_{i-1}, y_{i-1}) + 5f(x_{i-2}, y_{i-2})\right]$$

Eigen value:

Power Method : $\mathbf{v}^{(k+1)} = \frac{1}{m_{k+1}} A\mathbf{v}^{(k)}$, $k = 0,1,2,\ldots$

Shifted Power Method: $\mathbf{A}_{\text{shifted}} = \mathbf{A} - \lambda_{\text{Largest}}\mathbf{I}$, $\lambda_{\text{smallest}} = \lambda_{\text{Shifted}} + \lambda_{\text{Largest}}$

Gerschogorin's theorem:

$$r_i = \sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij}|, \qquad D_i = \{z \in \mathbf{C} : |z - a_{ii}| \leq r_i\}, \qquad \lambda_k \in \bigcup_{i=1}^{n} D_i \quad \text{for } k = 1,2,\ldots,n$$

FINAL EXAMINATION

SEMESTER/SESSION: SEM II / 2017/2018 PROGRAMME CODE: BWA
COURSE NAME : NUMERICAL ANALYSIS I COURSE CODE : BWA 20903

Integration:

$\frac{1}{3}$ Simpson's rule: $\int_a^b f(x)dx \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i \text{ even}}}^{n-2} f_i \right]$

Romberg method: $I\left(h, \frac{1}{2}h\right) = \frac{1}{3}\left[4I\left(\frac{1}{2}h\right) - I(h) \right]$,

where $I(h) = I_1$, $I\left(\frac{1}{2}h\right) = I_2$ and $I\left(h, \frac{1}{2}h\right) = I_3$.

Lagrange polynomial interpolation:

$P_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i), i = 0,1,2,\ldots,n$ where $L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{(x - x_j)}{(x_i - x_j)}$