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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2017/2018

COURSE NAME	:	MATHEMATICS FOR ENGINEERING TECHNOLOGY III
COURSE CODE	:	BWM 22403
PROGRAMME CODE	:	BND / BNE / BNF / BNG / BNH / BNL / BNM
EXAMINATION DATE	:	JUNE / JULY 2018
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS B) ALL CALCULATIONS MUST BE IN **3 DECIMAL** PLACES

THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

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Q1 (a) Given the function $z = f(x,y) = 2 - \sqrt{4 - x^2 - y^2}$.

(i) Sketch the level curves of the function by using values of $c = 0, 1, 1.5$ and 2.

(5 marks)

(ii) Hence, sketch the 3D graph of the function.

(2 marks)

(b) Find

$$\int_0^1 \int_0^{z^2} \int_0^3 y\cos(z^5)\ dxdydz.$$

(7 marks)

(c) Use triple integrals in cylindrical coordinates to determine the volume of the solid below $z = 3$, above the paraboloid $z = -(x^2 + y^2)$ and inside the cylinder $x^2 + y^2 = 9$.

(11 marks)

Q2 (a) Given the function $3x^2 = \sin(3x) - \ln(x+1) + 2$. By using secant method, compute the positive root in interval [0,1]. Iterate until $f(x_i) < 0.005$.

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(7 marks)

(b) A biologist has placed three strains of bacteria (denoted I, II and III) in a test tube, where they will feed on three different food sources (A, B and C). Each day 400 units of A, 700 units of B and 500 units of C are placed in the test tube. Each bacteria consumes a certain number of units of each food per day, as shown in the **Table Q2(b)** below

Table Q2(b): Bacteria Strains and The Food Sources

	Bacteria Strain I	Bacteria Strain I	Bacteria Strain I
Food A	5	1	0
Food B	0	1	2
Food C	1	3	1

Form a system of linear equations based on the above problem. Hence, calculate the number of bacteria of each strain that can coexist in the test tube and consume all of the food by using

(i) Gauss elimination method.

(9 marks)

(ii) Gauss-Seidel iteration method. Use initial guess $(74 \quad 31 \quad 335)^T$ and iterate until $\max\left\{\left|x_i^{(k+1)} - x_i^{(k)}\right|\right\} < 0.005$.

(9 marks)

Q3 (a) A tutoring service has kept records of performance on a standardized test and the number of days students attend their review classes as in **Table Q3(a)**. The performance rating Y represents the percent improvement in the test score students attain after taking the exam a second time. X is the number of attendance days in the review class.

Table Q3(a): Record of Attendance Days and Performance in Test

X, attendance days	1	2.5	5	6.5	9
Y, % improvement	2	5	11	14	17

By assuming $Y = f(X)$ is the true function relating X and Y, use Newton's divided difference method to estimate $f(4)$, that is the % improvement in one's score after 4 days of attending review classes.

(10 marks)

(b) Given the function $g(x) = e^x + 1$. Compute $\int_{1.5}^{4.5} \frac{1}{x} g(x)\, dx$ by using

(i) $\frac{3}{8}$ Simpson's rule with 6 subinterval.

(10 marks)

(ii) 2-point Gauss quadrature method.

(5 marks)

Q4 (a) Given the matrix

$$A = \begin{pmatrix} 0 & 11 & -5 \\ -2 & 17 & -7 \\ -4 & 26 & -10 \end{pmatrix}.$$

Calculate the smallest eigenvalue and the corresponding eigenvector by using inverse power method. Use $\mathbf{v}^{(0)} = (0.4 \quad 1 \quad 1)^T$.

(12 marks)

(b) The temperature distribution $T(x,t)$ of one dimensional silver rod is governed by the heat equation

$$\frac{\partial T}{\partial t} = \alpha^2 \frac{\partial^2 T}{\partial x^2}$$

with $\alpha^2 = 1.71$ is thermal diffusity. Given the initial condition

$$T(x,0) = \begin{cases} x, & 0 \le x \le 2 \\ 4 - x, & 2 \le x \le 4 \end{cases}$$

and the boundary conditions

$$T(0,t) = t, \ T(4,t) = t^2.$$

Approximate the temperature distribution of the rod with $\Delta x = h = 1$ and $\Delta t = k = 0.2$ for $0 \le t \le 0.4$ by using explicit finite difference method.

(13 marks)

- END OF QUESTIONS -

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Formulas

Multiple integrals

Cylindrical coordinate : $x = r\cos\theta$, $y = r\sin\theta$, $z = z$, $x^2 + y^2 = r^2$, $0 \le \theta \le 2\pi$.

$$\iiint_G f(x,y,z)\,dV = \iiint_G f(r,\theta,z)\,r\,dz\,dr\,d\theta$$

Nonlinear equations

Secant method : $x_{i+2} = \dfrac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}, i = 0,1,2,\ldots$

System of linear equations

Gauss-Seidel iteration : $x_i^{(k+1)} = \dfrac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}}{a_{ii}}$, $\forall i = 1, 2, 3, .., n$.

Interpolation

Newton divided difference :

$$P_n(x) = f_0^{[0]} + f_0^{[1]}(x - x_0) + f_0^{[2]}(x - x_0)(x - x_1) + \ldots + f_0^{[n]}(x - x_0)(x - x_1)\ldots(x - x_{n-1})$$

Numerical integration

Simpson $\frac{3}{8}$ rule : $\displaystyle\int_a^b f(x)\,dx \approx \frac{3}{8}h\left[\begin{matrix}(f_0 + f_n) + 3(f_1 + f_2 + f_4 + f_5 + \cdots + f_{n-2} + f_{n-1}) \\ +2(f_3 + f_6 + \cdots + f_{n-3})\end{matrix}\right]$

Gauss quadrature : For $\displaystyle\int_a^b f(x)\,dx$, $x = \dfrac{(b-a)t + (b+a)}{2}$

2-points: $\displaystyle\int_{-1}^{1} f(x)\,dx \approx g\left(-\frac{1}{\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right)$

Eigenvalue

Power Method : $v^{(k+1)} = \dfrac{1}{m_{k+1}} A v^{(k)}$, $k = 0,1,2,\ldots$,

Inverse Power Method : $\lambda_{\text{smallest}} = \dfrac{1}{\lambda_{\text{Shifted}}}$

Partial differential equations

Heat equation- Finite difference method:

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \qquad \frac{u_{i,j+1} - u_{i,j}}{k} = c^2 \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2}$$