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**UNIVERSITI TUN HUSSEIN ONN MALAYSIA**

**FINAL EXAMINATION  
SEMESTER II  
SESSION 2017/2018**

COURSE NAME : LINEAR ALGEBRA  
COURSE CODE : BWA 10303  
PROGRAMME CODE : BWA/BWQ  
EXAMINATION DATE : JUNE/JULY 2018  
DURATION : 3 HOURS  
INSTRUCTION : ANSWER ALL QUESTIONS

THIS QUESTION PAPER CONSISTS OF **FOUR (4)** PAGES

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Q1 Let  $A = \begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & p & q \end{pmatrix}$  and  $|A| = 4$ . Find the following determinants:

(a)  $\begin{pmatrix} a & b & c & d \\ 2e & 2f & 2g & 2h \\ i & j & k & l \\ 3m & 3n & 3p & 3q \end{pmatrix}$ .

(3 marks)

(b)  $\begin{pmatrix} e & g & f & h \\ a & c & b & d \\ i & k & j & l \\ m & p & n & q \end{pmatrix}$ .

(3 marks)

(c)  $\begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m+2a-5i & n+2b-5j & p+2c-5k & q+2d-5l \end{pmatrix}$ .

(3 marks)

(d)  $\begin{pmatrix} a & b & a & c & d \\ e & f & b & g & h \\ 0 & 0 & 1 & 0 & 0 \\ i & j & c & k & l \\ m & n & d & p & q \end{pmatrix}$ .

(3 marks)



Q2 Let  $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & \alpha & 2 \\ 1 & 1 & \alpha(\alpha+1) \end{bmatrix}$ . Find the value(s) of  $\alpha$  so that  $A$  has rank.

(a) 3.

(5 marks)

(b) 2.

(1 marks)

(c) 1.

(2 marks)

**Q3** (a) Which of this statement is not true

- (i) If  $A$  is symmetric,  $A^k$  is symmetric (1 mark)
- (ii) If  $A$  is skew-symmetric,  $A^k$  is symmetric if  $k$  is even (1 mark)
- (iii) If  $A$  is symmetric,  $A^k$  is skew symmetric if  $k$  is even (1 mark)
- (iv) If  $A^k$  is skew-symmetric,  $A$  is symmetric if  $k$  is odd (1 mark)

(b) Let  $A$ ,  $B$  and  $I$  be any square matrices, then

- (i) If  $A$  and  $B$  is symmetric,  $AB$  is symmetric if and only if  $AB = BA$  (1 mark)
- (ii)  $A^T + A$  is symmetric,  $A^T - A$  and  $A - A^T$  is skew-symmetric (1 mark)
- (iii)  $I$  is both symmetric and skew-symmetric (1 mark)
- (iv)  $A$  can be written uniquely as the sum of a symmetric and skew-symmetric matrix (1 mark)

**Q4** Factorize the determinant  $\begin{vmatrix} x + y & y + z & z + x \\ x & y & z \\ y & y & x \end{vmatrix}$ . (7 marks)



**Q5** Let  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ .

- (a) Show that if  $a \neq 0$  then the matrix  $A$  has a unique  $LU$ -decomposition with 1's along the main diagonal of  $L$ . Hence, write  $A = LU$ . (6 marks)
- (b) Show that if  $a = d = 0$  and  $b = c = 1$ , then  $A$  is invertible but has no  $LU$  decomposition. (5 marks)

**Q6** Solve the system of linear equations

$$\begin{aligned} x + y &= 3 \\ 2x + y + z &= 7 \\ x + 2y + z &= 8 \end{aligned}$$

- (a) By Cramer's rule. (5 marks)
- (b) By Gauss-Jordan reduction (5 marks)
- (c) By finding the inverse of the coefficient matrix (8 marks)

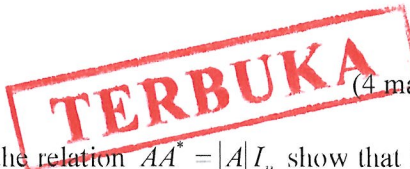
**Q7** Let  $A = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$

- (a) Find the eigenvalues of  $A$  and the eigenvectors for each eigenvalue. (10 marks)
- (b) Find a nonsingular matrix  $P$  such that  $A$  is diagonalizable. (5 marks)
- (c) Using the result in (b), find  $A^5$ . (5 marks)

**Q8** (a) Given that  $|A^{-1}| = 2$ , find  $|A^{-2}|$  and  $|A^{100}|$ . (4 marks)

(b) Given that  $|A| = -2$ , and  $|B| = 5$ , find  $|AB'|$ . (4 marks)

(c) Let  $A$  be a square matrix of order  $n$ . Form the relation  $AA^* = |A|I_n$ , show that if  $A$  is invertible, then  $|A^*| = |A|^{n-1}$ . (8 marks)



– END OF QUESTIONS –