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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2017/2018

COURSE NAME	:	LINEAR ALGEBRA
COURSE CODE	:	BWA 10303
PROGRAMME CODE	:	BWA/BWQ
EXAMINATION DATE	:	JUNE/JULY 2018
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **FOUR (4)** PAGES

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Q1 Let $A = \begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & p & q \end{pmatrix}$ and $|A| = 4$. Find the following determinants:

(a) $\begin{pmatrix} a & b & c & d \\ 2e & 2f & 2g & 2h \\ i & j & k & l \\ 3m & 3n & 3p & 3q \end{pmatrix}$.

(3 marks)

(b) $\begin{pmatrix} e & g & f & h \\ a & c & b & d \\ i & k & j & l \\ m & p & n & q \end{pmatrix}$.

(3 marks)

(c) $\begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m+2a-5i & n+2b-5j & p+2c-5k & q+2d-5l \end{pmatrix}$.

(3 marks)

(d) $\begin{pmatrix} a & b & a & c & d \\ e & f & b & g & h \\ 0 & 0 & 1 & 0 & 0 \\ i & j & c & k & l \\ m & n & d & p & q \end{pmatrix}$

(3 marks)

Q2 Let $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & \alpha & 2 \\ 1 & 1 & \alpha(\alpha+1) \end{bmatrix}$. Find the value(s) of α so that A has rank.

(a) 3.

(5 marks)

(b) 2.

(1 marks)

(c) 1.

(2 marks)

Q3 (a) Which of this statement is not true

(i) If A is symmetric, A^k is symmetric

(1 mark)

(ii) If A is skew-symmetric, A^k is symmetric if k is even

(1 mark)

(iii) If A is symmetric, A^k is skew symmetric if k is even

(1 mark)

(iv) If A^k is skew-symmetric, A is symmetric if k is odd

(1 mark)

(b) Let A, B and I be any square matrices, then

(i) If A and B is symmetric, AB is symmetric if and only if $AB = BA$

(1 mark)

(ii) $A^T + A$ is symmetric, $A^T - A$ and $A - A^T$ is skew-symmetric

(1 mark)

(iii) I is both symmetric and skew-symmetric

(1 mark)

(iv) A can be written uniquely as the sum of a symmetric and skew-symmetric matrix

(1 mark)

Q4 Factorize the determinant $\begin{vmatrix} x+y & y+z & z+x \\ x & y & z \\ y & y & x \end{vmatrix}$.

(7 marks)

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Q5 Let $= \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

(a) Show that if $a \neq 0$ then the matrix A has a unique LU-decomposition with 1's along the main diagonal of L. Hence, write $A = LU$.

(6 marks)

(b) Show that if $a = d = 0$ and $b = c = 1$, then A is invertible but has no LU decomposition.

(5 marks)

Q6 Solve the system of linear equations

$$x + y = 3$$
$$2x + y + z = 7$$
$$x + 2y + z = 8$$

(a) By Crammer's rule.

(5 marks)

(b) By Gauss-Jordan reduction

(5 marks)

(c) By finding the inverse of the coefficient matrix

(8 marks)

Q7 Let $A = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$

(a) Find the eigenvalues of A and the eigenvectors for each eigenvalue.

(10 marks)

(b) Find a nonsingular matrix P such that A is diagonalizable.

(5 marks)

(c) Using the result in (b), find A^5.

(5 marks)

Q8 (a) Given that $|A^{-1}| = 2$, find $|A^{-2}|$ and $|A^{100}|$.

(4 marks)

(b) Given that $|A| = -2$, and $|B| = 5$, find $|AB'|$.

(4 marks)

(c) Let A be a square matrix of order n. Form the relation $AA^* = |A| I_n$ show that if A is invertible, then $|A^*| = |A|^{n-1}$

(8 marks)

– **END OF QUESTIONS** –