UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2016/2017

COURSE NAME	:	CALCULUS OF VARIATION
COURSE CODE	:	BWA 31203
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JUNE 2017
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **THREE (3)** PAGES

Q1 Find an approximate solution for $y(x)$ for which the functional

$$J[y(x)]=\int_0^1\left(-y'^2+y^2-xy\right)dx, \qquad y(0)=0, \qquad y(1)=1,$$

is extremum, using finite difference approximation of order h, where h = 0.25. Next, show the error of approximation at x = 0.25, 0.50 and 0.75.

(20 marks)

Q2 (a) Determine the extremals for the following functional

$$J[y(x)]=\int_{t=0}^{t=\pi/2}\left(\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+2xy\right)dt,$$

subject to the boundary conditions

$$x(0)=0,\ y(0)=0,\ x\left(\frac{\pi}{2}\right)=1,\ y\left(\frac{\pi}{2}\right)=1.$$

[Hint: General solution for $\dfrac{d^4y}{dt^4}-m^2y(t)=0$ is

$y(t)=C_1e^{\sqrt{m}\,t}+C_2e^{-\sqrt{m}\,t}+C_3\sin\left(\sqrt{m}\,t\right)+C_4\cos\left(\sqrt{m}\,t\right)$]

(10 marks)

(b) Show that the extremal of the elementary variational problem can be included in the extremal field (proper or central).

$$J[y(x)]=\int_0^1\left(2e^xy+y'^2\right)dx, \quad y(0)=1, \quad y(1)=e.$$

(5 marks)

(c) If two batteries which have resistance r and electromotive force v are connected in parallel to a resistance R, then power loss in the resistance is given by

$$P=\frac{4v^2R}{(2r+R)^2}.$$

How much should be the resistance R be so that the power loss is maximum?

(5 marks)

Q3 (a) Find the shortest distance between circle $x^2+y^2=4$ and straight line $2x+y=6$.

(12 marks)

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(b) Show that for the functional

$$J[y]=\int_0^a \left(x^2+y^2+y'^2\right)\,dx, \quad u(0)=0 \text{ where } z=\delta y,$$

the Jacobi condition is fulfilled and for the extremal, $u=c\sinh(x)$, the functional is minimum where c is a constant.

(8 marks)

Q4 (a) Show that the extremum of the functional

$$J[y(x)]=\int_0^\pi \left(y'^2-y^2\right)dx, \quad y(0)=0, \quad y(\pi)=1,$$

subject to the constraint

$$K[y(x)]=\int_0^\pi y\,dx=1$$

is defined by a family $y=-\frac{1}{2}\cos x+\frac{1}{2}\left(1-\frac{\pi}{2}\right)\sin x+\frac{1}{2}$.

(10 marks)

(b) Find the general solution of the extremal for the functional

$$J[y(x)]=\int_0^1 y^2\left(y'^2-x^2\right)dx, \quad y(0)=0, \quad y(1)=1,$$

under coordinate transformation $x^2=u,\ y^2=v$.

(10 marks)

Q5 Solve the boundary value problem

$$y''+y=e^x, \quad y(0)=0, \quad y\left(\frac{\pi}{2}\right)=0,$$

by direct integration method and Ritz method. Compare the two results.

(20 marks)

- END OF QUESTIONS -