Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2015/2016

COURSE NAME	:	ENGINEERING MATHEMATICS III
COURSE CODE	:	BWM20403
PROGRAMME	:	BEJ / BEV
EXAMINATION DATE	:	JUNE / JULY 2016
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **FIVE (5) PAGES**

Q1 (a) Find the slope of the tangent line of the curve that is intersecting of the surface $z = x^2 - y^2$ and the plane $x = 2$, at the point (2,1,3).

(3 marks)

(b) Given formula $i = \dfrac{V}{R}$. From experiment, we obtained $V = 250$ volt and $R = 50$ ohm.

(i) Find the maximum error in calculating i if the error of value voltage, V is 1 volt and resistance, R is 0.5 ohm.

(ii) Find the maximum percentage of error in calculating the i if the maximum possible error of value voltage, V and resistance, R is 2% and 1%, respectively.

(12 marks)

(c) Find the local extremum of the function $f(x, y) = xy^2 - 6x^2 - 3y^2$.

(10 marks)

Q2 (a) By using double integrals, find the volume of the solid enclosed by planes $y = \sqrt{x}$ and $z = 1 - x$, in the first octant.

(5 mark)

(b) (i) By changing to cylindrical coordinates, evaluate $\displaystyle\int_0^2 \int_0^{\sqrt{4-y^2}} \int_0^{\sqrt{16-x^2-y^2}} z\,dz\,dx\,dy$

(ii) By using spherical coordinates, find the volume of the solid bounded above by sphere $x^2 + y^2 + z^2 = z$ and below by cone $z = \sqrt{x^2 + y^2}$.

(12 marks)

(c) A lamina which has density function $\rho(x, y) = y$ occupies the region bounded by $y = e^x$, $y = 0$, $x = 0$ and $x = 1$. Find:

(i) its mass by using formula, $m = \displaystyle\iint_R \rho(x, y)\,dA$.

(ii) its coordinate $\bar{y}$ of its center of mass.

(8 marks)

Q3 (a) Compute the curl of the vector field $\mathbf{F}(x, y, z) = e^{x+y}\mathbf{i} + \sin y\,\mathbf{j} + \cos^2 z\,\mathbf{k}$.

(3 marks)

(b) Use Green's Theorem to evaluate the line integral $\oint_C (x^2 - y)dx + x^2 dy$ where C is the boundary of the region between $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ oriented counterclockwise.

(10 marks)

(c) Given the force field $\mathbf{F}(x,y,z)=y\mathbf{i}+(x+2y)\mathbf{j}$

(i) Show that $\mathbf{F}$ is a conservative force field.

(3 mark)

(ii) Find a potential function ϕ such that $\mathbf{F}=\nabla\phi$.

(5 marks)

(iii) Hence, evaluate the work done by the force field $\mathbf{F}$ on a particle that moves along the curve C, where C is the upper semicircle that starts from (0, 1) to (1, 0).

(4 marks)

Q4 (a) State the Divergence Theorem and Stokes' Theorem.

(4 marks)

(b) If σ is the surface of sphere $x^2+y^2+z^2=4$ and $\mathbf{F}(x,y,z)=7x\mathbf{i}-z\mathbf{k}$:

(i) Find the divergence of F.

(3 marks)

(ii) Use Gauss's Theorem to evaluate $\iint_{\sigma}\mathbf{F}.\mathbf{n}dS$

(8 marks)

(c) Use the Stokes' Theorem to find the work performed by the force field

$$\mathbf{F}(x,y,z)=e^{z}\mathbf{i}+e^{z}\sin y\mathbf{j}+e^{z}\cos y\mathbf{k}$$

on a particle that oriented upward around the plane $z=y^2$ in the domain of $0\leq x\leq 4$ and $0\leq y\leq 2$.

(10 marks)

- END OF QUESTION -

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Second Derivative Test for Extreme of two variable functions

$G(x,y) = f_{xx}(x,y) f_{yy}(x,y) - \left(f_{xy}(x,y)\right)^2$

Case1: If $G(a,b) > 0$ and $f_{xx}(x,y) < 0$ then f has local maximum at (a,b)

Case2: If $G(a,b) > 0$ and $f_{xx}(x,y) > 0$ then f has local minimum at (a,b)

Case3: If $G(a,b) < 0$ then f has a saddle point at (a,b)

Case4: If $G(a,b) = 0$ then no conclusion can be made.

Polar coordinate

$x = r\cos\theta,\ \ y = r\sin\theta,\ \ \theta = \tan^{-1}(y/x)$, and $\iint\limits_R f(x,y)dA = \iint\limits_R f(r,\theta)\, r\, dr\, d\theta$

Cylindrical coordinate

$x = r\cos\theta,\ \ y = r\sin\theta,\ \ z = z$ and $\iiint\limits_G f(x,y,z)dV = \iiint\limits_G f(r,\theta,z)\, r\, dz\, dr\, d\theta$

Spherical coordinate

$x = \rho\sin\varphi\cos\theta,\ \ y = \rho\sin\varphi\sin\theta,\ \ z = \rho\cos\phi$, then $x^2 + y^2 + z^2 = \rho^2$, for $0 \le \theta \le 2\pi$, $0 \le \varphi \le \pi$, and $\iiint\limits_G f(x,y,z)dV = \iiint\limits_G f(\rho,\phi,\theta)\rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

$A = \iint\limits_R dA$

$m = \iint\limits_R \delta(x,y)dA$, where $\delta(x,y)$ is a density of lamina

$V = \iint\limits_R f(x,y)\,dA$

$V = \iiint\limits_G dV$

$m = \iiint\limits_G \delta(x,y,z)dV$

Formulas for curve in space

Let C is a smooth curve given by $\mathbf{r}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j} + z(t)\,\mathbf{k}$, t is parameter, then

Arc length of C in the interval [a, b], $s = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\,dt = \int_a^b \|\mathbf{r}'(t)\|\,dt$

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FORMULAS

If f is a differentiable function of x, y and z, then the

Gradient of f, grad $f(x, y, z) = \nabla f(x, y, z) = \dfrac{\partial f}{\partial x}\mathbf{i} + \dfrac{\partial f}{\partial y}\mathbf{j} + \dfrac{\partial f}{\partial z}\mathbf{k}$

Directional derivatives of f in the direction of a unit vector u, $D_u f(x, y, z) = \nabla f(x, y, z) \cdot \mathbf{u}$

If $\mathbf{F}(x, y, z) = M\,\mathbf{i} + N\,\mathbf{j} + P\,\mathbf{k}$ is a vector field, then the

Divergence of $\mathbf{F}(x, y, z)$, div $\mathbf{F} = \nabla \cdot \mathbf{F} = \dfrac{\partial M}{\partial x} + \dfrac{\partial N}{\partial y} + \dfrac{\partial P}{\partial z}$

Curl of $\mathbf{F}(x, y, z)$, curl $\mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\dfrac{\partial P}{\partial y} - \dfrac{\partial N}{\partial Z}\right)\mathbf{i} - \left(\dfrac{\partial P}{\partial x} - \dfrac{\partial M}{\partial Z}\right)\mathbf{j} + \left(\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}\right)\mathbf{k}$

F is conservative vector field if Curl of $\boldsymbol{F} = 0$.

Line Integral

$$\int_C f(x, y, z)\, ds = \int_a^b f(x(t), y(t), z(t))\,|\,r'(t)\,|\, dt$$

$$\int \mathrm{F}\cdot d\mathrm{r} = \int_a^b < M, N, P > \cdot < x'(t), y'(t), z'(t) > dt$$

Green's Theorem oriented counterclock-wise

$$\oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$$

Surface Integral

Let S be a surface with equation $z = g(x, y)$ and let R be its projection on the xy-plane.

$$\iint_S f(x, y, z)\, dS = \iint_R f(x, y, g(x, y))\sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dA$$

$$\iint_S \mathbf{F}\cdot\mathbf{n}\, dS = \iint_R \mathbf{F}\cdot\left[-\frac{\partial z}{\partial x}\mathbf{i} - \frac{\partial z}{\partial y}\mathbf{j} + \mathbf{k}\right] dA, \text{ oriented upward}$$

$$\iint_S \mathbf{F}\cdot\mathbf{n}\, dS = \iint_R \mathbf{F}\cdot\left[+\frac{\partial z}{\partial x}\mathbf{i} + \frac{\partial z}{\partial y}\mathbf{j} - \mathbf{k}\right] dA, \text{ oriented downward}$$

Gauss's Theorem

$$\iint_S \mathbf{F}\cdot\mathbf{n}\, dS = \iiint_G \nabla\cdot\mathbf{F}\, dV$$

Stokes' Theorem

$$\iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}\, dS = \int_C \mathbf{F}\cdot d\mathbf{r}$$