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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2015/2016

COURSE NAME	:	CALCULUS OF VARIATION
COURSE CODE	:	BWA 31203
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JUNE / JULY 2016
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL FIVE (5)** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **THREE (3)** PAGES

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Q1 (a) Given the functional, $J[y]=\int_0^1 \left(x^2 y'^2 + y^2\right)\, dx$.

(i) Find its first variation, $\delta J[y]$.

(5 marks)

(ii) Find its second variation, $\delta^2 J[y]$.

(4 marks)

(iii) Show that the functional is continuous on the function $y_0(x)=x$.

(4 marks)

(b) If two batteries which have resistance r and electromotive force v are connected in parallel to a resistance R, then the power loss in the resistance is given by

$$P=\frac{4v^2R}{(2r+R)^2}.$$

How much should the resistance R be so that the power loss is maximum?

(7 marks)

Q2 (a) Find the extremals for the following functional

$$J[y(x)]=\int_{-1}^{0}\left(240y - y'''^2\right)dx,$$

subject to the conditions

$$y(-1)=1,\ y(0)=0,\ y'(-1)=-4.5,$$
$$y'(0)=0,\ y''(-1)=16,\ y''(0)=0.$$

(10 marks)

(b) Show that there is no solution to the problem of finding a possible extremal to the functional

$$J[y(x)]=\int_0^1 \sqrt{y(x)-x}\ dx,$$

with boundary conditions $y(0)=0,\ y(1)=1$ and $y(x)\geq x$ on [0, 1].

(3 marks)

Q3 (a) Find the shortest distance from the point A (1, 0, -1) to the point B (0, -1, 1) lying on the surface $x+y+z=0$.

(14 marks)

(b) Determine whether the Jacobi condition is fulfilled for the extremal of the functional

$$J[y]=\int_{-1}^{1}\left(12xy+y'^2+x^2\right)dx$$

which extremal passes through the points O (-1, -2) and A (1, 0).

(7 marks)

(c) Utilizing the Legendre condition, test the following functional for extrema.

$$J[y]=\int_{0}^{1}\left(y'^2+x^2\right)dx, \quad y(0)=-1, \quad y(1)=1.$$

(6 marks)

Q4 Find the function that will extremize $J[y(x)]=\int_0^1\left(y'^2(x)+y(x)y'(x)\right)dx$ with boundary conditions $y(0)=1,\ y(1)=1$ when subject to the isoperimetric condition of

$$K[y(x)]=\int_0^1\left(y(x)-y'^2(x)\right)dx=1.$$

(20 marks)

Q5 By using the direct method of Ritz, find an approximate solution to the nonlinear equation $y''+x=0$ with boundary conditions $y(0)=1$ and $y(1)=0$ and compare it with the exact solution.

(20 marks)

- END OF QUESTION -