Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2015/2016

COURSE NAME	:	CALCULUS I
COURSE CODE	:	BWA 10203
PROGRAMME	:	1 BWA
EXAMINATION DATE	:	DECEMBER 2015/JANUARY 2016
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS.

THIS QUESTION PAPER CONSISTS OF SIX **(6)** PAGES

Q1 (a) Given

$$f(x)=\begin{cases} x-2, & x<0 \\ x^2, & x\geq 0 \end{cases}.$$

Find

(i) $\lim_{x\to 0} f(x)$.

(ii) $f(0)$.

(iii) Is the function continuous at $x=0$? Justify your answer.

(3 marks)

(b) Find the limits.

(i) $\lim_{x\to\infty}\left(\sqrt{x^2+3x}-\sqrt{x^2-2x}\right)$

(ii) $\lim_{x\to 0}\dfrac{\dfrac{1}{2+x}-\dfrac{1}{2}}{x}$

(iii) $\lim_{x\to 1}\dfrac{3x^3-3}{4x^3-x-3}$

(iv) $\lim_{x\to 0}\dfrac{3^{\sin x}-1}{x}$

(8 marks)

(c) Find the derivative of y with respect to x.

(i) $y=\log_8(\log_2 x)$

(ii) $\ln(x^2+y)=e^{x+2}$

(iii) $y=\sqrt{\dfrac{x}{x+1}}$

(9 marks)

Q2 (a) Given $y = e^x(\cosh x + \sinh x)$. Show that $\dfrac{d^2y}{dx^2} = 4y$

(5 marks)

(b) Given $f(x) = 9x^{\frac{2}{3}}$.

(i) Write the expression of $f'(x)$.

(ii) Find the value of x when $f(x) = 36$.

(iii) Hence, find the approximate value of x when $f(x)$ increases from 36.0 to 36.4.

(6 marks)

(c) Let $f(x) = (x^2 - 1)^3$.

(i) Find all critical points of $f(x)$.

(ii) Hence, determine whether the critical points is minimum, maximum or inflection point.

(9 marks)

Q3 (a) Evaluate these integrals.

(i) $\int x^2 \sqrt{1+2x}\, dx$

(ii) $\int \ln(x+2)\, dx$.

(iii) $\int_0^{\pi} \dfrac{\sin 2x}{2\cos x}\, dx$.

(iv) $\int_0^{\infty} x^2 e^{-x}\, dx$.

(12 marks)

(b) Given $y = x\sqrt{x+1}$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{3x+2}{2\sqrt{x+1}}$.

(ii) Hence, evaluate $\int_3^8 \dfrac{3x+2}{\sqrt{x+1}}\, dx$.

(8 marks)

Q4 (a) By using proper substitution, evaluate $\int \frac{x^2}{\sqrt[4]{x^3+2}}dx$.

(5 marks)

(b) Find $\int \frac{x^3+3}{4-x^2}dx$.

(5 marks)

(c) Show that $\int \frac{3x+5}{(x+1)(x-1)^2}dx = \frac{1}{2}\ln\left|\frac{x+1}{x-1}\right| + \frac{4}{x-1} + c$.

(10 marks)

Q5 (a) Find the derivative of y with respect to x.

(i) $y = \sinh^{-1}(\ln x)$.

(ii) $y = \ln(x^2+4) - x\tan^{-1}\left(\frac{x}{2}\right)$

(6 marks)

(b) Evaluate $\int \frac{dx}{\sqrt{4x^2-25}}$, $x > \frac{5}{2}$

(3 marks)

(c) Find area of the surface that is generated by revolving the curve $y = \sqrt[3]{3x}$ between $y=-1$ and $y=0$ about the y-axis.

(6 marks)

(d) Find the curvature if $x = \cos t$ and $y = \ln 2t$ at $t = \pi$.

(5 marks)

- END OF QUESTION –

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Formulae

Indefinite Integrals

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}+C, \quad n\neq -1$$

$$\int \frac{1}{x}\,dx = \ln|x|+C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \csc x \cot x\,dx = -\csc x + C$$

$$\int e^x\,dx = e^x + C$$

$$\int \cosh x\,dx = \sinh x + C$$

$$\int \sinh x\,dx = \cosh x + C$$

$$\int \mathrm{sech}^2 x\,dx = \tanh x + C$$

$$\int \mathrm{csch}^2 x\,dx = -\coth x + C$$

$$\int \mathrm{sech}\, x \tanh x\,dx = -\mathrm{sech}\, x + C$$

$$\int \mathrm{csch}\, x \coth x\,dx = -\mathrm{csch}\, x + C$$

Integration of Inverse Functions

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C, \ |x|<1$$

$$\int \frac{-1}{\sqrt{1-x^2}}\,dx = \cos^{-1} x + C, \ |x|<1$$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$$

$$\int \frac{-1}{1+x^2}\,dx = \cot^{-1} x + C$$

$$\int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \sec^{-1} x + C, \ |x|>1$$

$$\int \frac{-1}{|x|\sqrt{x^2-1}}\,dx = \csc^{-1} x + C, \ |x|>1$$

$$\int \frac{1}{\sqrt{x^2+1}}\,dx = \sinh^{-1} x + C$$

$$\int \frac{1}{\sqrt{x^2-1}}\,dx = \cosh^{-1} x + C, \ |x|>1$$

$$\int \frac{-1}{|x|\sqrt{1-x^2}}\,dx = \mathrm{sech}^{-1} |x| + C, \ 0<x<1$$

$$\int \frac{-1}{|x|\sqrt{1+x^2}}\,dx = \mathrm{csch}^{-1} |x| + C, \ x\neq 0$$

$$\int \frac{1}{1-x^2}\,dx = \begin{cases} \tanh^{-1} x + C, & |x|<1 \\ \coth^{-1} x + C, & |x|>1 \end{cases}$$

TRIGONOMETRIC SUBSTITUTION

Expression	*Trigonometry*	*Hyperbolic*
$\sqrt{x^2+k^2}$	$x = k\tan\theta$	$x = k\sinh\theta$
$\sqrt{x^2-k^2}$	$x = k\sec\theta$	$x = k\cosh\theta$
$\sqrt{k^2-x^2}$	$x = k\sin\theta$	$x = k\tanh\theta$

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TRIGONOMETRIC SUBSTITUTION

$t = \tan \frac{1}{2}x$		$t = \tan x$	
$\sin x = \dfrac{2t}{1+t^2}$	$\cos x = \dfrac{1-t^2}{1+t^2}$	$\sin 2x = \dfrac{2t}{1+t^2}$	$\cos 2x = \dfrac{1-t^2}{1+t^2}$
$\tan x = \dfrac{2t}{1-t^2}$	$dx = \dfrac{2dt}{1+t^2}$	$\tan 2x = \dfrac{2t}{1-t^2}$	$dx = \dfrac{dt}{1+t^2}$

IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

Trigonometric Functions

$$\cos^2 x + \sin^2 x = 1$$
$$\sin 2x = 2\sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$
$$1 + \tan^2 x = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$
$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$
$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$
$$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$
$$2\sin ax \cos bx = \sin(a+b)x + \sin(a-b)x$$
$$2\sin ax \sin bx = \cos(a-b)x - \cos(a+b)x$$
$$2\cos ax \cos bx = \cos(a-b)x + \cos(a+b)x$$

Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$
$$\cosh x = \frac{e^x + e^{-x}}{2}$$
$$\cosh^2 x - \sinh^2 x = 1$$
$$\sinh 2x = 2\sinh x \cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x = 2\cosh^2 x - 1 = 1 + 2\sinh^2 x$$
$$1 - \tanh^2 x = \text{sech}^2 x$$
$$\coth^2 x - 1 = \text{csch}^2 x$$
$$\tanh 2x = \frac{2\tanh x}{1 + \tanh^2 x}$$
$$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$$
$$\sinh(x \pm y) = \sinh x \cosh y \pm \sinh y \cosh x$$
$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

CURVATURE, ARC LENGTH AND SURFACE AREA OF REVOLUTION

$$\kappa = \frac{\left|\dfrac{d^2y}{dx^2}\right|}{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}$$

$$\kappa = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{\left[\dot{x}^2 + \dot{y}^2\right]^{3/2}}$$

$$L = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$L = \int_{y_1}^{y_2} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$S = 2\pi \int_{x_1}^{x_2} f(x) \sqrt{1 + \left(\frac{d}{dx}[f(x)]\right)^2}\, dx$$

$$S = 2\pi \int_{y_1}^{y_2} g(y) \sqrt{1 + \left(\frac{d}{dy}[g(y)]\right)^2}\, dy$$