UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2015/2016

COURSE NAME	:	STATISTICS FOR ENGINEERING TECHNOLOGY
COURSE CODE	:	BWM 22502
PROGRAMME	:	2BNN/ 2BNL/ 2BNB
EXAMINATION DATE	:	DECEMBER 2015/ JANUARY 2016
DURATION	:	2 HOURS 30 MINUTES
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS EXAMINATION PAPER CONSISTS OF **FIVE (5)** PAGES

Q1 (a) Let k be a constant and consider the probability distribution function

$$f(x) = \begin{cases} h(x - x^2), & 0 \le x \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate:

(i) The value of h
(ii) Expectation of X, $E(X)$
(iii) Variance of X, $Var(X)$
(iv) $E(2X-1)$
(v) $Var(2X-1)$

(15 marks)

(b) The number of fault per month that arise in the gear boxes of buses is known to follow a Poisson distribution with mean of 2.5 faults per month.

(i) Find the mean and variance of the faults.
(ii) What is the probability that in a given month no faults are found?

(5 marks)

Q2 (a) A machine used to extract juice from mangos obtains an amount from each mango that is approximately normally distributed with mean of 133.2 grams and standard deviation of 11.3 grams. Suppose that a sample of 30 mangos is selected.

(i) Calculate the probability that the mean extract juice is at least 130.4 grams.
(ii) If the sample increase to 50 mangos, find the probability that the mean extract juice is between 131.2 and 135.6 grams.

(11 marks)

(b) A process used in filling bottles with soft drink result in weight (in liter) that are normally distributed. There are two production lines that filling the soft drink in the bottles. The distribution for production Line 1 is $X_1 \sim N(2.2, 0.5^2)$ and for production Line 2 is $X_2 \sim N(2.3, 0.4^2)$. A random sample of 100 bottles in Line 1 and 150 bottles in Line 2 is selected. What is the probability that average weight filled in

(i) production Line 2 is at most 2.32 liter?
(ii) production Line 1 is 0.02 liter less than production Line 2?

(9 marks)

Q3 (a) A random sample of the number of farms (in thousands) in various states in Malaysia is given in **Table Q3(a)**.

Table Q3(a): Number of Farm (in thousand)

23	45	10	18	20	39
18	16	29	9	38	33

(i) Calculate the point estimate for the sample variance.

(ii) Construct the 90% confidence interval of variance for the number of farms.

(10 marks)

(b) Two groups of students are given a problem-solving test, and the results are shown in **Table Q3(b)**. Construct the 95% confidence interval for the ratio of the standard deviation for the two groups, $\frac{\sigma_1}{\sigma_2}$.

Table Q3(b): Problem Solving Test

	Group 1 : Finance Major	Group 2 : Management Major
Sample size	13	9
Variance	15.9	11.4

(10 marks)

Q4 (a) Two different formulations of an oxygenated motor fuel are being tested to study their road octane numbers. The population variance of road octane number for formulation 1 is 1.63 and for formulation 2 it is 1.40. Two random samples of size 15 and 20 are tested from formulation 1 and formulation 2. The mean road octane numbers observed are 81.55 and 83.14 for formulation 1 and formulation 2 respectively. Test whether formulation 2 produces a higher octane numbers than formulation 1 at 5% level of significant.

(10 marks)

(b) The sugar content of the syrup in canned peaches is normally distributed and the population variance is thought to be 18 $(mg)^2$. Test the hypothesis that the variance is not 18 $(mg)^2$ if a random sample of 10 cans yields a sample standard deviation of 4 mg, by using 0.01 level of significant.

(10 marks)

Q5 Zaiton wish to buy a car. She read a newspaper to find the price of the used car for a local compact car. The data of the age (in years) and the prices (RM in thousand) are shown in **Table Q5** below.

Table Q5: The Price of Used Local Compact Car

Age (x)	1	2	3	4	5	6	7	8	9	10	11	12
Price (y)	33.4	29.3	29.0	28.1	27.5	26.0	24.2	19.5	14.7	14.0	13.4	13.0

(a) Sketch a scatter plot for the data.

(2 marks)

(b) Use the method of least squares to estimate the regression line. Interpret the results.

(8 marks)

(c) Test the slope, $\beta_1 = -1$ at 5% level of significance.

(8 marks)

(d) Estimate the car price when the car is 14 years old.

(2 marks)

- END OF QUESTION –

STATISTICAL FORMULAE

$\sum_{i=-\infty}^{\infty} p(x_i) = 1$ $\quad \int_{-\infty}^{\infty} f(x)\, dx = 1$ $\quad \text{Var}(X) = E(X^2) - [E(X)]^2$

$E(X) = \sum_{\forall x} xp(x)$ $\quad E(X) = \int_{-\infty}^{\infty} xf(x)\, dx$

$E(X^2) = \sum_{\forall x} x^2 p(x)$ $\quad E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$

$p(x) = \binom{n}{x} p^x (1-p)^{n-x},\ x = 0, 1, 2, \ldots, n$ $\quad p(x) = \dfrac{e^{-\mu}\mu^x}{x!},\ x = 0, 1, 2, \ldots$

$X \sim N(\mu, \sigma^2)$ $\quad \bar{X} \sim N(\mu, \dfrac{\sigma^2}{n})$ $\quad Z = \dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

$Z \sim N(0, 1)$ $\quad Z = \dfrac{X-\mu}{\sigma}$

$\bar{X}_1 - \bar{X}_2 \sim N(\mu_1 - \mu_2, \dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2})$ $\quad Z = \dfrac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} \sim N(0, 1)$

$T = \dfrac{\bar{X}-\mu}{S/\sqrt{n}} \sim t_{\alpha, n-1}$ $\quad F = \dfrac{S_1^2}{S_2^2} \sim f_{\alpha, n_1-1, n_2-1}$

$\chi^2 = \dfrac{(n-1)S^2}{\sigma^2} \sim \chi^2_{\alpha, n-1}$ $\quad s^2 = \dfrac{1}{n-1}\left(\sum x^2 - \dfrac{(\sum x)^2}{n}\right)$

$T = \dfrac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{S_P\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}} \sim t_{\alpha,\ n_1+n_2-2}$ $\quad S_P^2 = \dfrac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}$

$T = \dfrac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}} \sim t_{\alpha, v}$ $\quad v = \dfrac{\left(S_1^2/n_1 + S_2^2/n_2\right)^2}{\dfrac{\left(S_1^2/n_1\right)^2}{(n_1-1)} + \dfrac{\left(S_2^2/n_2\right)^2}{(n_2-1)}}$ or $v = 2(n\text{-}1)$

$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$ $\quad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$ $\quad \text{SSE} = S_{yy} - \hat{\beta}_1 S_{xy}$ $\quad \text{MSE} = \dfrac{\text{SSE}}{n-2}$

$\hat{\beta}_1 = \dfrac{S_{xy}}{S_{xx}}$

$s_{xx} = \sum_{i=1}^{n} x_i^2 - \dfrac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}$ $\quad s_{yy} = \sum_{i=1}^{n} y_i^2 - \dfrac{\left(\sum_{i=1}^{n} y_i\right)^2}{n}$ $\quad s_{xy} = \sum_{i=1}^{n} x_i y_i - \dfrac{\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n}$

$\hat{\beta}_1 \sim N\left(\beta_1, \dfrac{\sigma^2}{S_{xx}}\right)$ $\quad T = \dfrac{\hat{\beta}_1 - \beta_1^*}{\sqrt{\text{MSE}/S_{xx}}} \sim t_{\alpha, n-2}$ $\quad r^2 = \dfrac{(S_{xy})^2}{S_{xx}S_{yy}}$

$\hat{\beta}_0 \sim N\left(\beta_0, \sigma^2\left(\dfrac{1}{n} + \dfrac{\bar{x}^2}{S_{xx}}\right)\right)$ $\quad T = \dfrac{\hat{\beta}_0 - \beta_0^*}{\sqrt{\text{MSE}\left(1/n + \bar{x}^2/S_{xx}\right)}} \sim t_{\alpha, n-2}$