CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2015/2016

COURSE NAME	:	RISK THEORY
COURSE CODE	:	BWA 40803
PROGRAMME	:	4 BWA
EXAMINATION DATE	:	DECEMBER 2015/JANUARY 2016
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SIX(6)** PAGES

Q1 (a) Assume that a decision maker's current wealth is RM10,000. Assign $u(0) = -1$ and $u(10,000) = 0$.

(i) When facing a loss of X with probability 0.5 and remaining at current wealth with probability 0.5, the decision maker would be willing to pay up to G for complete insurance. The values of X and G in three situations are given below **Table Q1(a)**:

Table Q1(a)

RM X	RM G
10,000	6000
6000	3300
3000	1700

Determine three values on the decision maker's utility of wealth function u.

(4 marks)

(ii) Calculate the slopes of the four line segments joining the five points determined on the graph $u(w)$. Determine the rates of changes of the slopes from segment to segment.

(3 marks)

(iii) Put yourself in the role of a decision maker with wealth RM10,000. In addition to the given values of $u(0)$ and $u(10,000)$, elicit three additional values on your utility of wealth function u.

(2 marks)

(iv) On the basis of the five values of your utility function, calculate the slopes as done in part Q1(a)(ii).

(3 marks)

(b) Amy faces the following wealth

$$\begin{pmatrix} 24 & 12 & 48 & 6 \\ \frac{2}{6} & \frac{3}{6} & \frac{1}{6} & 0 \end{pmatrix}$$

and tells you that she considers it equivalent to getting \$18.

(i) What is the risk premium associated with this lottery for Amy?

(4 marks)

(ii) Is Amy risk-loving, risk-neutral or risk-averse?

(4 marks)

Q2 (a) A fire insurance company covers 160 structures against fire damage up to an amount stated in the contract. The numbers of contracts at the different contract amounts are give below in **Table Q2(a)**:

Table Q2(a)

Contract Amount	Number of Contracts
10,000	80
20,000	35
30,000	25
50,000	15
100,000	5

Assume that for each of the structure, the probability of one claim within a year is 0.04, and the probability of more than one claim is 0. Assume that fire in the structures are mutually independent events. Furthermore, assume that the conditional distribution of the claim size, given that a claim has occurred, is uniformly distributed over the interval from 0 to the contract amount. Let N be the number of claims and let S be the amount of claims in a one-year period.

(i) Calculate the mean and variance of N.

(3 marks)

(ii) Calculate the mean and variance of S.

(4 marks)

(iii) What relative security loading, θ, should be used so that the company can collect an amount equal to the 99th percentile of the distribution of total claim? (Use a normal approximation).

(8 marks)

(b) Consider a portfolio of 32 policies. For each policy, the probability q of a claim 1/6 and B, the benefit amount given that there is a claim, has p.d.f

$$f(y) = \begin{cases} 2(1-y), & 0 < y < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Let S be the total claims for the portfolio. Using a normal approximation, estimate $P(S > 4)$.

(5 marks)

Q3 (a) Let $S = X_1 + X_2 + \cdots + X_N$, where $X_1, X_2, \ldots$ are independent, identically distributed random variables, and N is a random variable independent of the X_i's. Derive an expression for the moment generating function of S in terms of the probability generating function of N and the moment generating function of X_i.

(4 marks)

(b) In a group of policies, the monthly number of claims for a single policy has a Poisson distribution with parameter λ, where λ is a random variable with density $f(\lambda) = 2e^{-2\lambda}$, $\lambda > 0$.

(i) Show that the probability of n claims on a policy picked at random from the group is $\frac{2}{3^{n+1}}, n = 0, 1, 2, \ldots$

(5 marks)

(ii) Find the moment generating function for the aggregate claims distribution if the claims have a gamma distribution with mean 2 and variance 2.

(6 marks)

(c) The number of claims arising from a hurricane in a particular region has a Poisson distribution with mean λ. The claim severity distribution has mean 0.5 and variance 1.

(i) Determine the mean and variance of the total amount of claims arising from a hurricane.

(2 marks)

(ii) The number of hurricanes in this region in one year has a Poisson distribution with mean μ. Determine the mean and variance of the total amount claimed from all the hurricanes in this region in one year.

(3 marks)

Q4 Claims for a particular risk arrive in a Poisson process rate λ. The claim sizes are independent and identically distributed with density $f(x)$ and are independent of the claims arrival process. Assume there is a constant $\gamma(0 < \gamma < \infty)$ such that $lim_{r \to \gamma} M(r) = \infty$ where $M(r)$ is the moment generating function of a claim. Premium are received continuously at constant rate with premium loading factor $\theta > 0$.

(a) Define the adjustment coefficient, R.

(2 marks)

(b) Define the surplus process and the probability $\psi(u)$ of ruin with initial surplus $u > 0$.

(2 marks)

(c) Write down the Lundberg's inequality.

(2 marks)

(d) Derive the adjustment coefficient if $f(x) = \dfrac{1}{\mu} e^{\frac{x}{\mu}}, x > 0$, and $\theta = 0.25$.

(5 marks)

(e) Consider the case where $f(x) = \frac{1}{2} e^{-x}(1 + 2e^{-x}), x > 0$, and $\theta = 0.25$.

(i) Calculate the expected claim size μ.

(2 marks)

(ii) Calculate the corresponding adjustment coefficient, and determine an upper bound for $\psi(15)$.

(4 marks)

(iii) Compare your answers to Q4(e)(ii) with those obtained if the claim sizes are mistakenly assumed to be exponentially distributed with mean μ, and comment briefly.

(3 marks)

Q5 (a) The distribution of X which represents the claim severity from a portfolio of non-life insurance policies has a Pareto distribution with mean RM350 and standard deviation RM42. If the insurer arranges excess of loss reinsurance with retention RM1200. Calculate the probability that a claim will involve the reinsurer. [Hint: Pareto distribution: $p(x) = \dfrac{\alpha x_0^\alpha}{x^{\alpha+1}}$, mean $= \dfrac{\alpha x_0}{\alpha - 1}$ and Variance $= \dfrac{\alpha x_0^2}{(\alpha-2)(\alpha-1)^2}$]

(8 marks)

(b) Assume that S has a compound Poisson distribution with

$$\lambda = 1.5, \quad p(1) = \frac{2}{3}, \quad p(2) = 13.$$

Calculate the values of

$$f_S(x), \quad F_S(x), \quad E[I_x],$$

for $x = 0, 1, 2, 3, 4, 5,$ and 6.

(12 marks)

- END OF QUESTION -