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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2015/2016

COURSE NAME	:	ORDINARY DIFFERENTIAL EQUATIONS
COURSE CODE	:	BWA 20303
PROGRAMME	:	2 BWA
EXAMINATION DATE	:	DECEMBER 2015/JANUARY 2016
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL FIVE (5)** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

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Q1 Given

$$y'' = -y \text{ with } y(0) = 1 \text{ and } y'(0) = 0.$$

(a) By assuming $y = \sum_{0}^{\infty} c_m x^m$, show that the differential equation above can be expressed as

$$\sum_{n=0}^{\infty} [(n+2)(n+1)c_{n+2} + c_n]x^n = 0.$$

(6 marks)

(b) Show that the recurrence relation is given by

$$c_{n+2} = -\frac{c_n}{(n+1)(n+2)}, \quad n = 0, 1, 2, 3, \ldots$$

(2 marks)

(c) Deduce the coefficient of series c_n, for $n = 0, 1, 2, 3, \ldots$ in terms of c_0 and c_1.

(6 marks)

(d) Verify that the solution of the differential equation is

$$y(x) = \cos x.$$

(6 marks)

Q2 Given the system of first order differential equations

$$y'_1 = 4y_1 + 2y_2,$$
$$y'_2 = 3y_1 + 3y_2.$$

(a) Write the equation in matrix form $Y' = AY$ where A is the coefficient matrix.

(2 marks)

(b) Show that the eigenvalues are $\lambda_1 = 6$ and $\lambda_2 = 1$.

(5 marks)

(c) Find the corresponding eigenvectors of the eigenvalues found in **Q2(b)**.

(6 marks)

(d) Determine whether the corresponding eigenvectors are linearly independent or not.

(4 marks)

(e) Verify that the general solution is given by

$$y(x) = C_1\begin{pmatrix}1\\1\end{pmatrix}e^{6x} + C_2\begin{pmatrix}1\\-1.5\end{pmatrix}e^{x}.$$

(3 marks)

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Q3 (a) I have a cheese burger and a mug of hot Nescafe for my lunch break. The Nescafe is at $190^0 F$. The room temperature is $70^0 F$. At time $t = 0$, the Nescafe is cooling at $15^0 F$ per minute

Newton's law of cooling states that the rate at which the temperature, $T(t)$ changes in a cooling body is proportional to the difference between the temperature of the body and the constant temperature, T_0 of the surrounding medium.

(i) Model the equation of the cooling Nescafe.

(4 marks)

(ii) Determine the time for the temperature to reach $143^0 F$.

(6 marks)

(a) A spring is stretched 0.49 m $(\Delta\ell)$ when a 6 kg mass (m) is attached. The weight is then pulled down an additional 0.8 m and released with an upward velocity of 10 ms^{-1}. Neglect the damping constant, c. If the general equation describing the spring-mass system is

$$m\frac{d^2y}{dt^2} + c\frac{dy}{dt} + ky = 0,$$

construct an equation for the position of the spring at any time t.

(Hints : Weight, $W = mg$, $k = \dfrac{W}{\Delta\ell}$, $g \approx 9.8\,\text{ms}^{-2}$)

(10 marks)

Q4 (a) Find the general solution for the second order differential equation by the variation of parameters method.

$$y'' - 3y' + 2y = \frac{e^x}{1+e^x}$$

(10 marks)

(Hint : $\dfrac{1}{1+e^x} = \dfrac{e^{-x}}{1+e^{-x}}$, $\quad \dfrac{1}{e^x(1+e^x)} = \dfrac{1}{e^x} - \dfrac{1}{1+e^{-x}}$)

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(b) Consider the function

$$f(t)=\begin{cases} t-2 & , \quad 0\le t<4 \\ 2 & , \quad 4\le t<6 \\ 0 & , \quad t\ge 6 \end{cases}$$

(i) Sketch the graph of $f(t)$.

(ii) Write the function $f(t)$ in the form of unit step function.

(iii) Find the Laplace transform of $f(t)$.

(10 marks)

Q5 (a) Find

(i) $L^{-1}\left\{\dfrac{1}{s^2+s-2}\right\}$,

(4 marks)

(ii) $L^{-1}\left\{\dfrac{2}{s(s+2)(s-1)}\right\}$.

(4 marks)

(b) By using Laplace transform, solve

$$x' = -x + y,$$
$$y' = 2x,$$

subject to $x(0)=0, \quad y(0)=1$.

(12 marks)

– END OF QUESTION –

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FORMULA

Second-order Differential Equation

The roots of characteristic equation and the general solution for differential equation $ay''+by'+cy=0$ or $a\ddot{y}+b\dot{y}+cy=0$ or $a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=0$.

Characteristic equation: $am^2+bm+c=0$.		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y=Ae^{m_1x}+Be^{m_2x}$
2.	Real and equal roots: $m=m_1=m_2$	$y=(A+Bx)e^{mx}$
3.	Complex roots: $m_1=\alpha+\beta i$, $m_2=\alpha-\beta i$	$y=e^{\alpha x}(A\cos\beta x+B\sin\beta x)$

The method of undetermined coefficients

For non-homogeneous second order differential equation $ay''+by'+cy=f(x)$, the particular solution is given by $y_p(x)$:

$f(x)$	$y_p(x)$
$P_n(x)=A_nx^n+A_{n-1}x^{n-1}+\cdots+A_1x+A_0$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(P\cos\beta x+Q\sin\beta x)$
$P_n(x)e^{\alpha x}$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)e^{\alpha x}$
$P_n(x)\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)\cos\beta x+$ $x^r(C_nx^n+C_{n-1}x^{n-1}+\cdots+C_1x+C_0)\sin\beta x$
$Ce^{\alpha x}\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^re^{\alpha x}(P\cos\beta x+Q\sin\beta x)$
$P_n(x)e^{\alpha x}\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)e^{\alpha x}\cos\beta x+$ $x^r(C_nx^n+C_{n-1}x^{n-1}+\cdots+C_1x+C_0)e^{\alpha x}\sin\beta x$

Note : r is the least non-negative integer (r = 0, 1, or 2) which determine such that there is no terms in particular integral $y_p(x)$ corresponds to the complementary function $y_c(x)$.

The method of variation of parameters

If the solution of the homogeneous equation $ay''+by'+cy=0$ is $y_c=Ay_1+By_2$, then the particular solution for $ay''+by'+cy=f(x)$ is

$$y=y_c+y_p,\quad \text{and}\quad y_p=uy_1+vy_2,$$

$$\text{where } u=-\int\frac{y_2f(x)}{aW}dx,\qquad v=\int\frac{y_1f(x)}{aW}dx \quad\text{and}\quad W=\begin{vmatrix}y_1 & y_2\\ y_1' & y_2'\end{vmatrix}=y_1y_2'-y_2y_1'.$$

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Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
e^{at}	$\frac{1}{s-a}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\sin at$	$\frac{a}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\cos at$	$\frac{s}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$\int_0^t f(u)g(t-u)du$	$F(s)\cdot G(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$y(t)$	$Y(s)$
$t^n,\ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$y'(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$
$t^n f(t),\ n=1,2,3,\ldots$	$(-1)^n \frac{d^n}{ds^n}F(s)$		

Representation of Functions in Power Series

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots..$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots..$$

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