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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATTION
SEMESTER I
SESSION 2015/2016

COURSE NAME	:	OPTIMIZATION TECHNIQUES II
COURSE CODE	:	BWA 40703
PROGRAMME	:	4 BWA
EXAMINATION DATE	:	DECEMBER 2015 / JANUARY 2016
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

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Q1 (a) Consider the general constrained problem,

$$\begin{array}{ll} \text{Minimize} & f(x) \\ \text{subject to} & h(x)=0 \end{array}$$

with the penalty function is given by

$$P(x)=\frac{1}{2}h(x)^{\mathrm{T}}\Gamma h(x)$$

where Γ is a symmetric positive define $m\times m$ matrix. Let

$$q(c,x)=f(x)+cP(x),$$

find the first-order necessary conditions and the Hessian of $q(c,x)$.

(8 marks)

(b) Consider the constrained problem

$$\begin{array}{ll} \text{Minimize} & f(x)=x_1^2+10x_2^2 \\ \text{subject to} & x_1+x_2=4 \end{array}$$

and the penalty function is $P(x)=(h(x))^2$.

(i) Define the function $q(c,x)$.

(2 marks)

(ii) Find the gradient $\nabla q(c,x)$.

(3 marks)

(iii) By setting $\nabla q(c,x)=0$, show that

$$x_1=\frac{40c}{10+11c}, \qquad x_2=\frac{4c}{10+11c}.$$

(4 marks)

(iv) Evaluate x_1, x_2, $f(x)$ and $q(c,x)$ for $c=1$, 10, 100, 1000. Write your answer in a table form.

(8 marks)

Q2 (a) Consider the problem,

$$\begin{array}{ll} \text{Minimize} & f(x) \\ \text{subject to} & x \in S \end{array}$$

where the constraint set S has a nonempty interior that is arbitrarily close to any point of S, and this kind of set takes the form

$$S = \{x : g_i(x) \le 0, i = 1, 2, \dots, p\}.$$

Then, the function

$$B(x) = -\sum_{i=1}^{p} \frac{1}{g_i(x)},$$

defined on the interior of S, is a barrier function.

(i) Write the barrier objective function $r(c_k, x)$ and the corresponding necessary conditions, which is satisfied by the minimum point x_k.

(4 marks)

(ii) Show that the Hessian $R(c_k, x)$ of $r(c_k, x)$ is

$$R(c_k, x) = L(x) - \frac{1}{c_k} \sum_{i=1}^{p} \frac{2}{g_i(x)^3} \nabla g_i(x)^{\mathrm{T}} \nabla g_i(x).$$

(5 marks)

(b) Consider the problem,

$$\begin{array}{ll} \text{Minimize} & f(x) = 2x_1^2 + 9x_2 \\ \text{subject to} & x_1 + x_2 \ge 4. \end{array}$$

(i) Define the barrier objective function $r(c_k, x)$.

(2 marks)

(ii) Prove that the stationary point is given by $x_1 = \frac{9}{4}$, $x_2 = \frac{7}{4} - \frac{1}{3\sqrt{c}}$ when $\nabla r(c_k, x) = 0$.

(10 marks)

(iii) Interpret the optimal solution as c approaches infinity.

(4 marks)

Q3 (a) Consider the primal problem,

$$\begin{aligned} &\text{Minimize} && f(x)=\frac{1}{2}x^{\mathrm{T}}Qx \\ &\text{subject to} && Bx-b\le 0\,. \end{aligned}$$

(i) Write the dual function $\phi(\mu),\ \mu\ge 0,$ for the problem.

(2 marks)

(ii) Show that $x^{*}=-Q^{-1}B^{\mathrm{T}}\mu$ satisfies the necessary conditions.

(4 marks)

(iii) By substituting the optimal point x^{*} into the dual function $\phi(\mu)$, define a quadratic programming problem such that if this problem is solved for μ, that μ will be the Lagrange multiplier for the primal problem.

(6 marks)

(b) Consider the problem in two variables

$$\begin{aligned} &\text{Minimize} && -xy \\ &\text{subject to} && (x-3)^2+y^2=5 \end{aligned}$$

(i) Define the Lagrange function and obtain the first-order necessary conditions.

(3 marks)

(ii) Obtain the solutions x and y in terms of λ.

(6 marks)

(iii) Find the Hessian matrix of the corresponding Lagrange function for $\lambda=1$.

(2 marks)

(iv) Give your interpretation on the solution.

(2 marks)

Q4 (a) Consider the quadratic program

$$\begin{array}{ll} \text{Minimize} & \frac{1}{2}x^{\mathrm{T}}Qx + c^{\mathrm{T}}x \\ \text{subject to} & Ax = b \end{array}$$

where x is n-dimensional and b is m-dimensional. Prove the following proposition that gives conditions under which the system is nonsingular.

Proposition. *Let Q and A be $n \times n$ and $m \times n$ matrices, respectively. Suppose that A has rank m and that Q is positive definite on the subspace $M = \{x : Ax = 0\}$. Then the matrix*

$$\begin{bmatrix} Q & A^{\mathrm{T}} \\ A & 0 \end{bmatrix}$$

is nonsingular.

(8 marks)

(b) The augmented Lagrangian for the equality constrained problem,

$$\begin{array}{ll} \text{Minimize} & f(x) \\ \text{subject to} & h(x) = 0 \end{array}$$

is the function

$$l_c(x, \lambda) = f(x) + \lambda^T h(x) + \frac{1}{2}c\,|\,h(x)|^2.$$

(i) It is given that $f(x, y) = 2x^2 + 2xy + y^2 - 2y$ and $h(x, y) = x$, write the corresponding augmented Lagrangian $l_c(x, y, \lambda)$.

(3 marks)

(ii) Find the minimum point (x, y) of this problem. Give your answer in terms of c and λ.

(10 marks)

(iii) The iterative process for λ_k is

$$\lambda_{k+1} = \lambda_k - \frac{c(2 + \lambda_k)}{2 + c}.$$

Discuss the rate of convergence for the value of λ for any $c > 0$.

(4 marks)

– END OF QUESTION –