UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2015/2016

COURSE NAME	:	NUMERICAL ANALYSIS II
COURSE CODE	:	BWA 30403
PROGRAMME	:	3BWA
EXAMINATION DATE	:	DECEMBER 2015 / JANUARY 2016
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER **ALL** QUESTIONS **IN PART A** AND **TWO (2)** QUESTIONS **IN PART B.** 2. ALL CALCULATIONS AND ANSWERS MUST BE IN THREE (3) DECIMAL PLACES.

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

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PART A

Q1 (a) Solve the boundary value problem of

$$e^x y'' + xy' - 5(1+x)y = x^3, \quad 0 \le x \le 2,$$

with the following boundary conditions $y(0)=2, y(2)=5$ at $x=0(1.0)2$ by using shooting method, initially using fourth order Runge Kutta method.

(15 marks)

(b) Solve $\frac{d^2y}{dx^2} + y = 0$ using fourth order Runge-Kutta method for $x=0.2$ and $x=0.4$ correct to three decimal places. Initial conditions are $x=0$, $y=1$, and $y'=0$.

(10 marks)

Q2

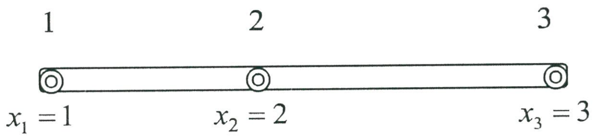

Figure Q2

Consider a fin of length 5 unit that has three nodes and two elements, as shown in **Figure Q2**. The heat flow equation is

$$\frac{d}{dx}\left(A(x)k(x)\frac{dT(x)}{dx}\right) + Q(x) = 0, \text{ for } \quad 0 \le x \le 3,$$

with $A(x)$ is the cross-sectional area, $k(x)$ is the thermal conductivity, $T(x)$ is the temperature at length x and $Q(x)$ is the heat supply per unit time and per unit length.

Find the temperature at each nodal points T_2, T_3 and $-\left(k\frac{dT}{dx}\right)_{at\,t=1}$, if $A(x)$ is 10 units, $k(x)$ is 5 units and $Q(x)$ is 50 units. Let the temperature at $x=0$ is 0 unit and the heat flux $-\left(k\frac{dT}{dx}\right)_{at\,t=3}$ is 20 units.

(25 marks)

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PART B

Q3 (a) Given the boundary value problem

$$y'' + xy' - 2y + x = 0, \quad 0 \le x \le 1,$$

with the boundary conditions $y'(0) = 1$ and $2y'(1) - y(1) = 3$. Derive the system of linear equations in matrix-vector form by finite difference method (**DO NOT SOLVE THE SYSTEM**). Use $\Delta x = h = 0.2$.

(10 marks)

(b) Derive the implicit finite difference Crank-Nicolson formula.

(5 marks)

(c) Given the heat equation

$$u_t = \frac{1}{9} u_{xx}, \quad 0 < x < 1, \ t > 0,$$

with conditions $u(0,t) = 1$, $u(1,t) = 1$ and $u(x,0) = \cos 2\pi x$. By using explicit finite-difference method, find the approximate solution to the heat equation for $x = 0$ to $x = 1$ and $t \le 0.01$. Take $\Delta x = 0.25$ and $\Delta t = 0.01$.

(10 marks)

Q4 (a) Fin the volume of the solid, V under the surface $z = 3x^3 + 3x^2 y$ over the rectangle $R = \{(x, y) : 0 \le x \le 3, 0 \le y \le 2\}$, using 2-point Gaussian quadrature.

(10 marks)

(b) Define Fourier and Fast Fourier Transform

(3 marks)

(c) Given that the wave equation $u_{tt} = 4u_{xx}$, $0 < x < 1$, $0 < t < 0.5$, with the boundary conditions $u(0,t) = u(1,t) = 0, 0 \le t \le 0.5$ and initial conditions

$$u(x,0) = \begin{cases} \dfrac{5x}{2} & for\ 0 \le x < 5\dfrac{3}{5}, \\ \dfrac{15 - 15x}{4} & for\ \dfrac{3}{5} \le x \le 1 \end{cases}$$

and $u_t(x,0) = 0$ for $0 \le x \le 1$. By taking $h = \Delta x = 0.2$ and $k = \Delta t = 0.1$, solve for u using finite difference method.

(12 marks)

Q5 (a) State and prove Lax Equivalence Theorem for convergent.

(7 marks)

(b) Use finite difference method with Gauss-Seidel iteration method and initial guess, $u^{(0)} = 0$ to approximate the solution to the the Poisson equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = x, 0 < x < 1, \quad 0 < y < 1,$$

with the boundary conditions

$$u(0, y) = 0, u(1, y) = \frac{1}{6}, 0 \leq y \leq 1,$$

$$u(x, 0) = \frac{1}{6}x^3, u(x, 1) = \frac{1}{6}x^3, 0 \leq x \leq 1,$$

with $h = \frac{1}{3}$ and $k = \frac{1}{3}$.

(18 arks)

- END OF QUESTION -

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FORMULAS

Gauss-Seidel iteration method:

$$x_i^{(k+1)} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}}{a_{ii}}, i = 1, 2, \ldots, n.$$

Numerical integration

$\frac{1}{3}$ Simpson's rule: $\int_a^b f(x)dx \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i \text{ even}}}^{n-2} f_i \right].$

$\frac{3}{8}$ Simpson's rule:

$$\int_a^b f(x)dx \approx \frac{3}{8}h\left[f_0 + f_n + 3(f_1 + f_2 + f_4 + f_5 + \ldots + f_{n-2} + f_{n-1}) + 2(f_3 + f_6 + \ldots + f_{n-6} + f_{n-3})\right]$$

2-point Gauss Quadrature: $\int_{-1}^{1} f(x)dx = c_1 f(x_1) + c_2 f(x_2) \approx g\left(-\sqrt{\frac{3}{5}}\right) + g\left(\sqrt{\frac{3}{5}}\right).$

Ordinary differential equations

Initial – Value Problem:

Fourth-order Runge-Kutta Method

$y_{i+1} = y_i + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$ where

$k_1 = h\, f(x_i, y_i),\quad k_2 = h\, f\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right),$

$k_3 = h\, f\left(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right)$ and $k_4 = h\, f(x_i + h, y_i + k_3),$

Boundary value problems:

Finite difference method:

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$$y_i' \approx \frac{y_{i+1} - y_{i-1}}{2h}, \qquad y_i'' \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}.$$

Partial differential equations

Poisson equation-Finite difference method

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j} + \left(\frac{\partial^2 u}{\partial y^2}\right)_{i,j} = f_{i,j} \qquad \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = f_{i,j}.$$

Finite element method

$$KT = F_b - F_l$$

where $K_{ij} = \int_p^q A(x)k(x)\frac{dN_i}{dx}\frac{dN_j}{dx}dx$ is stiffness matrix,

$$T = T_i,$$

$$F_b = \left[N_i A(x)k(x)\frac{dT}{dx}\right]_p^q,$$

$$F_l = -\int_p^q N_i Q(x)dx.$$