Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2014/2015

COURSE NAME	:	INTRODUCTION TO FUZZY SET THEORY
COURSE CODE	:	BWA 20603
PROGRAMME	:	2 BWA
EXAMINATION DATE	:	JUNE 2015/ JULY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN PART A AND **THREE (3)** QUESTIONS IN PART B

THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

PART A

Q1 (a) What is the role of $\alpha - cuts$ in fuzzy set theory and what is the difference between $\alpha - cuts$ and strong $\alpha - cuts$?

(6 marks)

(b) Given a set of five experts identified by their name $\{Ahmad, Baba, Chong, Danial, Elia\}$, who are involved in the survey to express their opinions on some policy issue. Given that the fuzzy relation Q expressed on this set by the matrix

$Q =$

	A	B	C	D	E
A	1	.8	.7	1	.9
B	.8	1	.7	.8	.8
C	.7	.7	1	.7	.7
D	1	.8	.7	1	.9
E	.9	.8	.7	.9	1

is an equivalence relations. Find the people who have the degree of similarity of opinions on the issue by showing the diagram connection from each node and equivalence classes in $\alpha - cuts$ of Q.

(14 marks)

Q2 (a) Perform the following operations in intervals

(i) [-1, 2] + [1, 3] (ii) [-1, 2] - [1, -3]

(iii) [-3, 4] × [-3, 4] (iv) [-4, 6] / [1, 2]

(4 marks)

(b) Let A defined on a universe of three discrete temperatures, $X = \{x_1, x_2, x_3\}$ and let B defined on a universe of two discrete pressures, $Y = \{y_1, y_2\}$. Fuzzy set A represents the "ambient temperature" and Fuzzy set B represents the "near optimum pressure" for a certain heat exchanger as:

$$A = \left\{\frac{0.2}{x_1}, \frac{0.5}{x_2}, \frac{1}{x_3}\right\} \text{ and } B = \left\{\frac{0.3}{y_1}, \frac{0.9}{y_2}\right\}$$

Find the *Cartesian product* represent the conditions temperature-pressure pairs of the exchager that is associated with "efficient operations".

(4 marks)

(c) Given that $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$ and $Z = \{z_1, z_2, z_3\}$. Consider the following relations $R = X \circ Y$, $S = Y \circ Z$ as below:

$$R = \begin{array}{c} \\ x_1 \\ x_2 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.7 & 0.5 \\ 0.8 & 0.4 \end{bmatrix} \end{array} \quad \text{and} \quad S = \begin{array}{c} \\ y_1 \\ y_2 \end{array} \begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} 0.9 & 0.6 & 0.5 \\ 0.1 & 0.7 & 0.5 \end{bmatrix} \end{array}$$

(i) Find the *max-product* composition of $T = R \circ S$.

(6 marks)

(ii) Find the *max-min product* composition of $T = R \circ S$.

(6 marks)

PART B

Q3 (a) Let X be the universe of military aircraft of intrest , as defined here:

$$X = \{A010, B052, B117, C005, F004, F014, F015, F016, F111, K130\}$$

Let A be the fuzzy set of bomber class aircraft:

$$A = \left\{ \frac{0.2}{F016} + \frac{0.4}{F004} + \frac{0.5}{A010} + \frac{0.5}{F014} + \frac{0.6}{F015} + \frac{0.8}{F111} + \frac{1.0}{B117} + \frac{1.0}{B052} \right\}$$

Let B be the fuzzy set of fighter class aircraft:

$$B = \left\{ \frac{0.1}{B117} + \frac{0.3}{F111} + \frac{0.5}{F004} + \frac{0.8}{F015} + \frac{0.9}{F014} + \frac{1.0}{F016} \right\}$$

Find the following various set combinations for these sets:

(i) $A \cdot B$ (ii) A / B

(ii) $(A \cup B)^C$ (iv) $A^C \cap B^C$

(8 marks)

(b) A factory process control operation involves two linguistic (atomic) parameters consisting of pressure and temperature in a fluid delivery system. Nominal pressure limits range from 400 *psi* maximum. Nominal temperature limits are 130^0F to 140^0F. We characterize each parameter in fuzzy linguistic terms as follows:

$$\text{"Low" temperature} = \left\{\frac{1.0}{131} + \frac{0.8}{132} + \frac{0.6}{133} + \frac{0.4}{134} + \frac{0.2}{135} + \frac{0.0}{136}\right\}$$

$$\text{"High" temperature} = \left\{\frac{0.0}{400} + \frac{0.2}{600} + \frac{0.4}{700} + \frac{0.6}{800} + \frac{0.8}{900} + \frac{1.0}{1000}\right\}$$

$$\text{"Low" pressure} = \left\{\frac{1.0}{400} + \frac{0.8}{600} + \frac{0.6}{700} + \frac{0.4}{800} + \frac{0.2}{900} + \frac{0.0}{1000}\right\}$$

Define the membership function for the following three linguistic expressions.

(i) Temperature not very low.

(3 marks)

(ii) Temperature very very high.

(3 marks)

(iii) Temperature not very low and not very high.

(6 marks)

Q4 A fuzzy tolerance relation R is reflexive and symmetric. Find the transitive closure, R_T of R by using three iterations.

$$R = \begin{bmatrix} 1 & 1 & 0 & 0.8 & 0.9 & 0 \\ 1 & 1 & 0.8 & 0.9 & 0.5 & 0 \\ 0 & 0.8 & 1 & 0 & 0 & 0.8 \\ 0.8 & 0.9 & 0 & 1 & 1 & 0 \\ 0.9 & 0.5 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0.8 & 0 & 0 & 1 \end{bmatrix}$$

(20 marks)

Q5 Let A and B be two fuzzy numbers whose membership functions are given by

$$A(x)=\begin{cases}\dfrac{x+2}{2} & for \quad -2<x\le 0\\ \dfrac{2-x}{2} & for \quad 0<x\le 2\\ 0 & otherwise\end{cases}$$

and

$$B(x)=\begin{cases}\dfrac{x-2}{2} & for \quad 2<x\le 4\\ \dfrac{6-x}{2} & for \quad 4<x\le 6\\ 0 & otherwise\end{cases}$$

(a) Sketch the graph for A and B in the same region.

(4 marks)

(b) Find $\alpha-cut$ for A and B respectively.

(4 marks)

(c) Calculate the fuzzy numbers $A+B$, $A-B$, $A\cdot B$, and A/B when $\alpha=0.5$.

(12 marks)

Q6 Fuzzy sets A, B and C are defined on the universe $X=[0,9]$ where

$$\mu_A=\frac{x}{x+1}, \quad \mu_B=2^{-x}, \quad \mu_C=(1+10(x-4))^{-1}.$$

(a) Calculate the coordinates of each membership functions.

(6 marks)

(b) Graphs of the membership functions in the same region.

(4 marks)

(c) Find

(i) μ_A^c.

(ii) $\mu_{A\cup B}$ and $\mu_{B\cap C}$.

(iii) $\mu_{A\cup B\cup C}$ and $\mu_{A\cap B\cap C}$.

(10 marks)

- END OF QUESTION –