UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2014/2015

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS I
COURSE CODE	:	BFC 13903/ BWM 10103/BSM1913
PROGRAMME	:	1 BFF/ 2 BFF/ 3 BFF/ 4 BFF
EXAMINATION DATE	:	DECEMBER 2014/JANUARY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

Q1 (a) State the domain and range of the following relation:
{(-3,5), (-2,5), (-1,5), (0,5), (1,5), (2,5)}

(2 marks)

(b) Evaluate each of the following limits by using L'Hopital's rule:

i) $\lim_{w \to -4} \frac{\sin(\pi w)}{w^2 - 16}$.

(6 marks)

ii) $\lim_{w \to 4} \frac{2-\sqrt{x}}{4-x}$.

(6 marks)

(d) Determine whether the given function are continuous at $x = 2$.

$$f(x) = \begin{cases} x^3 + 2 & \text{if} \quad x < 1 \\ 5 & \text{if} \quad x = 2 \\ x^2 + 6 & \text{if} \quad x > 2 \end{cases}$$

(6 marks)

Q2 (a) Find $f'(x)$ if $f(x) = e^{\ln(\cos 4x)} + \ln(x^2 - 3x) - \sin(\pi x)$.

(5 marks)

(b) By using the **natural logarithm (ln)**, find $\frac{dy}{dx}$ of $y = \frac{x^7 \sin(3x)}{e^{x^3}\sqrt{2x^5 - 5}}$.

(5 marks)

(c) By using the **Chain Rule**, find $\frac{du}{dv}$ if $u = t + \frac{1}{t}$ and $t = 1 - \frac{1}{v}$.

(5 marks)

(d) By using the **Implicit Differentiation**, find $\frac{dy}{dx}$ for $x^2 + \frac{1}{3}y^3 + \cos(xy) = 2y - 3$ at $x = \frac{\pi}{4}$ and $y = 2$.

(5 marks)

Q3 (a) A ball is thrown up vertically with the initial velocity of 20 *m/s* from ground level. The motion of the ball is given by

$$s = 10t - 2t^2.$$

Find
(i) the velocity at t=3,
(ii) the time of the ball to reach the maximum point,
(iii) the maximum attained by the ball,
(iv) the total time before the ball reaches the ground again,
(v) the velocity of the ball when it hits the ground.

(10 marks)

(b) Given a function $f(x) = \dfrac{x^2+1}{x}, \quad x \in \mathbf{R}, x \neq 0.$

Show that the graph of f have critical points and determine whether the point is a maximum or a minimum. Hence sketch the graph of $f(x)$.

(10 marks)

Q4 (a) Integrate $\int \sin^2 x \cos^3 x \, dx$.

(5 marks)

(b) Use substitute $t = \tan\left(\dfrac{x}{2}\right)$ and $\tan x = \dfrac{2t}{1-t^2}$ to integrate $\displaystyle\int \frac{dx}{\cos x + 1}$.

(5 marks)

(c) Evaluate $\displaystyle\int_0^1 \frac{dx}{\left(x^2+1\right)^{\frac{3}{2}}}$.

(6 marks)

(d) Evaluate the following integrals.

(i) $\displaystyle\int \frac{1}{2+9x^2}\,dx$

(ii) $\displaystyle\int \frac{dx}{x\sqrt{1+x^2}}$

(4 marks)

Q5 (a) Differentiate the following expressions with respect to x.

(i) $y = \cosh^{-1} x + \sin^{-1} x$

(ii) $y = \sec^{-1} x \csc^{-1} x$

(iii) $y = \dfrac{(2x^2 + 3)}{\cosh(3x - 2)}$

(8 marks)

(b) The parametric equation of a semi-circle is given as $x = a\cos\theta$ and $y = a\sin\theta$, where $0 \le \theta \le 2\pi$. Find the arc length of the semi-circle.

(5 marks)

(c) Find the radius of curvature of $y = \dfrac{4x}{x^2 + 3}$ at the points (0, 0) and (3, 3).

(7 marks)

- END OF QUESTIONS -

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Formulae

Trigonometric	Hiperbolic
$\cos^2 x + \sin^2 x = 1$	$\sinh x = \dfrac{e^x - e^{-x}}{2}$
$1 + \tan^2 x = \sec^2 x$	$\cosh x = \dfrac{e^x + e^{-x}}{2}$
$\cot^2 x + 1 = \text{cosec}^2 x$	$\cosh^2 x - \sinh^2 x = 1$
$\sin 2x = 2\sin x \cos x$	$1 - \tanh^2 x = \text{sech}^2 x$
$\cos 2x = \cos^2 x - \sin^2 x$	$\coth^2 x - 1 = \text{cosech}^2 x$
$\cos 2x = 2\cos^2 x - 1$	$\sinh 2x = 2\sinh x \cosh x$
$\cos 2x = 1 - 2\sin^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$	$\cosh 2x = 2\cosh^2 x - 1$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	$\cosh 2x = 1 + 2\sinh^2 x$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$
$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$
$2\sin x \cos y = \sin(x+y) + \sin(x-y)$	$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$
$2\sin x \sin y = -\cos(x+y) + \cos(x-y)$	$\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$
$2\cos x \cos y = \cos(x+y) + \cos(x-y)$	

Logarithm	Inverse Hiperbolic
$a^x = e^{x \ln a}$	$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$, any x.
$\log_a x = \dfrac{\log_b x}{\log_b a}$	$\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right)$, $x \geq 1$
	$\tanh^{-1} x = \dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$, $-1 < x < 1$

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$\frac{d}{dx}[k]=0$, k constant	$\int k\,dx = kx + C$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1}+C$, $n \neq -1$
$\frac{d}{dx}[\ln\lvert x\rvert]=\frac{1}{x}$	$\int \frac{dx}{x} = \ln\lvert x\rvert + C$
$\frac{d}{dx}[\cos x]=-\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}[\sin x]=\cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}[\tan x]=\sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}[\cot x]=-\text{cosec}^2 x$	$\int \text{cosec}^2 x\,dx = -\cot x + C$
$\frac{d}{dx}[\sec x]=\sec x\tan x$	$\int \sec x\tan x\,dx = \sec x + C$
$\frac{d}{dx}[\text{cosec}\,x]=-\text{cosec}\,x\cot x$	$\int \text{cosec}\,x\cot x\,dx = -\text{cosec}\,x + C$
$\frac{d}{dx}[e^x]=e^x$	$\int e^x\,dx = e^x + C$
$\frac{d}{dx}[\cosh x]=\sinh x$	$\int \sinh x\,dx = \cosh x + C$
$\frac{d}{dx}[\sinh x]=\cosh x$	$\int \cosh x\,dx = \sinh x + C$
$\frac{d}{dx}[\tanh x]=\text{sech}^2 x$	$\int \text{sech}^2 x\,dx = \tanh x + C$
$\frac{d}{dx}[\coth x]=-\text{cosech}^2 x$	$\int \text{cosech}^2 x\,dx = -\coth x + C$
$\frac{d}{dx}[\text{sech}\,x]=-\text{sech}\,x\tanh x$	$\int \text{sech}\,x\tanh x\,dx = -\text{sech}\,x + C$
$\frac{d}{dx}[\text{cosech}\,x]=-\text{cosech}\,x\coth x$	$\int \text{cosech}\,x\coth x\,dx = -\text{cosech}\,x + C$

Integration of Inverse Functions

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$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C.$
$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C.$
$\int \frac{dx}{\lvert a\rvert\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C.$
$\int \frac{dx}{\sqrt{x^2 + a^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C, \qquad a > 0.$
$\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C, \qquad x > a$
$\int \frac{dx}{x^2 - a^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C, & \lvert x\rvert < a \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C, & \lvert x\rvert > a \end{cases}$
$\int \frac{dx}{x\sqrt{a^2 - x^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{x}{a}\right) + C, \qquad 0 < x < a.$
$\int \frac{dx}{x\sqrt{a^2 + x^2}} = -\frac{1}{a}\operatorname{cosech}^{-1}\left(\frac{x}{a}\right) + C, \qquad 0 < x < a.$

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y	$\frac{dy}{dx}$
$\sin^{-1} u$	$\frac{1}{\sqrt{1-u^2}}.\frac{du}{dx}, \qquad \lvert u \rvert < 1.$
$\cos^{-1} u$	$-\frac{1}{\sqrt{1-u^2}}.\frac{du}{dx}, \qquad \lvert u \rvert < 1.$
$\tan^{-1} u$	$\frac{1}{1+u^2}.\frac{du}{dx}$
$\cot^{-1} u$	$-\frac{1}{1+u^2}.\frac{du}{dx}$
$\sec^{-1} u$	$\frac{1}{\lvert u \rvert\sqrt{u^2-1}}.\frac{du}{dx}, \qquad \lvert u \rvert > 1.$
$\text{cosec}^{-1} u$	$-\frac{1}{\lvert u \rvert\sqrt{u^2-1}}.\frac{du}{dx}, \qquad \lvert u \rvert > 1.$
$\sinh^{-1} u$	$\frac{1}{\sqrt{u^2+1}}.\frac{du}{dx}$
$\cosh^{-1} u$	$\frac{1}{\sqrt{u^2-1}}.\frac{du}{dx}, \qquad \lvert u \rvert > 1.$
$\tanh^{-1} u$	$\frac{1}{1-u^2}.\frac{du}{dx}, \qquad \lvert u \rvert < 1.$
$\coth^{-1} u$	$-\frac{1}{1-u^2}.\frac{du}{dx}, \qquad \lvert u \rvert > 1.$
$\text{sech}^{-1} u$	$-\frac{1}{u\sqrt{1-u^2}}.\frac{du}{dx}, \qquad 0 < u < 1.$
$\text{cosech}^{-1} u$	$-\frac{1}{\lvert u \rvert\sqrt{1+u^2}}.\frac{du}{dx}, \qquad u \neq 0.$