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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2014/2015

COURSE NAME	:	ENGINEERING MATHEMATICS V/ STATISTICS ENGINEERING
COURSE CODE	:	BEE 31702/ BWM 20502
PROGRAMME	:	2/3/4 BEJ, 2/3/4 BEV, 1/2/3/4 BDD
EXAMINATION DATE	:	JUNE 2015 /JULY 2015
DURATION	:	2 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

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Q1 (a) Define the random variable and list the two types of random variable.

(3 marks)

(b) Suppose the range of a discrete random variable is $\{0, 1, 2, 3, 4\}$. If the probability distribution function is $f(x) = cx$ for $x = 0, 1, 2, 3, 4$, estimate the value of c.

(3 marks)

(c) Given probability density function for continuous random variable X as below where k is a constant.

$$f(x) = \begin{cases} k^2 & , \quad 0 \le x < 2 \\ 2k^2x - 3k^2 & , \quad 2 \le x < 3 \\ 0 & , \quad \text{otherwise} \end{cases}$$

(i) Find the value of k.

(3 marks)

(ii) Calculate $P(X>2.1)$.

(3 marks)

(iii) Calculate $P(2.8<X<3.1)$.

(3 marks)

(d) An average of three cars arrives at highway tollage every minute. If this rate is approximated by a Poisson process, calculate the probability that:

(i) at least five cars will arrive in two-minute period?

(2 marks)

(ii) not more than seven cars will arrive in three-minute period?

(3 marks)

(e) If the random variable X follows a normal distribution with mean, $\mu = 25$ and variance, σ^2, and $P(X < 28.5) = 0.7734$. Find the value of σ.

(5 marks)

Q2 (a) Intravenous fluid bags are filled by an automated filling machine. Assume that the fill volumes of the bags are normally distributed with a standard deviation of 0.10 fluid ounces.

(i) Determine the standard deviation of the sampling distribution of the mean fill volume of 25 bags.

(2 marks)

(ii) If the mean fill volume of the machine is 6.16 ounces, calculate the probability that the mean fill volume of 25 bags is more than 6.10 fluid once ounces.

(3 marks)

(iii) If the mean fill volume of the machine is 6.16 ounces, determine the probability that the mean fill volume of 25 bags is between 6.18 and 6.20 fluid ounces.

(3 marks)

(iv) If the mean fill volume of the machine is 6.16 ounces, compute the probability that the mean fill volume of 25 bags is less than 6.14 fluid ounces.

(3 marks)

(b) State the definition of population and population distribution.

(4 marks)

(c) A company manufactures two types of cables, brand A and brand B that have mean breaking strengths of 4000 kg and 4500 kg and standard deviations of 300 kg and 200 kg, respectively. There are 100 cables of brand A and 50 cables of brand B are tested.

(i) State the sampling distribution for mean brand A.

(3 marks)

(ii) Calculate the probability that the mean breaking strengths of brand B will be at least 600 kg more than brand A.

(7 marks)

Q3 (a) A food manufacturer produces peanut butter of various brand. 15 jars of a certain brand of peanut butter are selected at random. The mean percentage of impurities for the sample is 2.213 with a standard deviation of 0.584. Compute the 99% confidence interval for the mean number of the percentage of impurities in 15 jars of a certain brand of peanut butter. Assume that the variable is approximately normally distributed.

(5 marks)

(b) Pull-strength tests on 10 soldered leads for a semiconductor device yield the following results in kg-force required to rupture the bond:

15.8 12.7 13.2 16.9 10.6 18.8 11.1 14.3 17.0 12.5

Another set of 8 leads was tested after encapsulation to determine whether the pull strength has been increased by encapsulation of the device, with the following results:

24.9 23.6 19.8 22.1 20.4 21.6 21.8 22.5

Assume that the populations are approximately normal distributed with unequal variances. Determine the 95% confidence interval of the difference between two samples.

(12 marks)

(c) A computer simulation program generates exponentially distributed random variables of unknown mean. 200 samples of these random variables are generated and grouped into 10 batches of 20 samples each. The sample means of the 10 batches are given in Table **Q3(c)**.

Table Q3(c): Raw data of 10 batches

1.04	0.64	0.81	0.76	1.12
1.30	0.98	0.65	1.39	1.27

Determine the 95% confidence interval of the variance.

(8 marks)

Q4 A rocket motor is manufactured by bonding together two types of propellants, an igniter and a sustainer. The shear strength of the bond y is thought to be linear function of the age of the propellant x when the motor is cast. Ten observations are shown in the following **Table Q4.**

Table Q4: Strength of bond y against age of propellant x

Observation Number	Strength, y (psi)	Age, x (weeks)
1	2158	15
2	1678	23
3	2316	8
4	2061	17
5	2207	5
6	1708	19
7	1784	24
8	2575	2
9	2357	7
10	2277	11

(a) Assuming a linear relationship, use the least squares method to find the simple linear regression model and interpret the meaning of β_1.

(15 marks)

(b) Determine the approximate number of strength of bond if the age of propellant is 27.

(2 marks)

(c) Determine the value of Pearson correlation. Interpret the result.

(5 marks)

(d) Determine the value of coefficient of determination.

(3 marks)

- END OF QUESTIONS –

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Formula

Special Probability Distributions :

$$P(X=r)=\frac{e^{-\lambda}\cdot\lambda^r}{r\,!},\ r=0,1,\ldots,\infty,\ X\sim P_0(\lambda),\ Z=\frac{X-\mu}{\sigma},\ Z\sim N(0,1),\ X\sim N(\mu,\sigma^2).$$

Sampling Distributions :

$$\bar{X}\sim N(\mu,\sigma^2/n),\ Z=\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}\sim N(0,1),\ T=\frac{\bar{x}-\mu}{s/\sqrt{n}},\ \bar{X}_1-\bar{X}_2\sim N\left(\mu_1-\mu_2,\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}\right).$$

Estimations :

$$n=\left(\frac{Z_{\alpha/2}\cdot\sigma}{E}\right)^2,\ \bar{x}\pm z_{\alpha/2}\left(\sigma/\sqrt{n}\right),\ \bar{x}\pm z_{\alpha/2}\left(s/\sqrt{n}\right),\ \bar{x}\pm t_{\alpha/2,v}\left(\frac{s}{\sqrt{n}}\right)$$

$$\left(\bar{x}_1-\bar{x}_2\right)-Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}<\mu_1-\mu_2<\left(\bar{x}_1-\bar{x}_2\right)+Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}},$$

$$\left(\bar{x}_1-\bar{x}_2\right)-Z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}<\mu_1-\mu_2<\left(\bar{x}_1-\bar{x}_2\right)+Z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}},$$

$$\left(\bar{x}_1-\bar{x}_2\right)-t_{\alpha/2,v}\cdot S_p\sqrt{\frac{2}{n}}<\mu_1-\mu_2<\left(\bar{x}_1-\bar{x}_2\right)+t_{\alpha/2,v}\cdot S_p\sqrt{\frac{2}{n}};v=2n-2$$

$$\left(\bar{x}_1-\bar{x}_2\right)-t_{\alpha/2,v}\cdot S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}<\mu_1-\mu_2<\left(\bar{x}_1-\bar{x}_2\right)+t_{\alpha/2,v}\cdot S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

where Pooled estimate of variance, $S_p^2=\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$ with $v=n_1+n_2-2$,

$$\left(\bar{x}_1-\bar{x}_2\right)-t_{\alpha/2,v}\sqrt{\frac{1}{n}\left(s_1^2+s_2^2\right)}<\mu_1-\mu_2<\left(\bar{x}_1-\bar{x}_2\right)+t_{\alpha/2,v}\sqrt{\frac{1}{n}\left(s_1^2+s_2^2\right)}\ \text{with}\ v=2(n-1),$$

$$\left(\bar{x}_1-\bar{x}_2\right)-t_{\alpha/2,v}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}<\mu_1-\mu_2<\left(\bar{x}_1-\bar{x}_2\right)+t_{\alpha/2,v}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} \text{ with } v=\frac{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1}+\frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2-1}},$$

$$\frac{(n-1)\cdot s^2}{\chi^2_{\alpha/2,v}}<\sigma^2<\frac{(n-1)\cdot s^2}{\chi^2_{1-\alpha/2,v}} \text{ with } v=n-1,$$

$$\frac{s_1^2}{s_2^2}\cdot\frac{1}{f_{\alpha/2}(v_1,v_2)}<\frac{\sigma_1^2}{\sigma_2^2}<\frac{s_1^2}{s_2^2}\cdot f_{\alpha/2}(v_2,v_1) \text{ with } v_1=n_1-1 \text{ and } v_2=n_2-1.$$

Simple Linear Regressions :

$$S_{xy}=\sum x_i y_i-\frac{\sum x_i\cdot\sum y_i}{n},\ S_{xx}=\sum x_i^2-\frac{\left(\sum x_i\right)^2}{n},\ S_{yy}=\sum y_i^2-\frac{\left(\sum y_i\right)^2}{n},\ \bar{x}=\frac{\sum x}{n},\ \bar{y}=\frac{\sum y}{n},$$

$$\hat{\beta}_1=\frac{S_{xy}}{S_{xx}},\ \hat{\beta}_0=\bar{y}-\hat{\beta}_1\bar{x},\ \hat{y}=\hat{\beta}_0+\hat{\beta}_1 x,\ r=\frac{S_{xy}}{\sqrt{S_{xx}\cdot S_{yy}}},\ SSE=S_{yy}-\hat{\beta}_1 S_{xy},\ MSE=\frac{SSE}{n-2},$$