UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2014/2015

COURSE NAME	:	PARTIAL DIFFERENTIAL EQUATION
COURSE CODE	:	BWA30303
PROGRAMME	:	3 BWA
EXAMINATION DATE	:	DECEMBER 2014/ JANUARY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL FIVE (5)** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) Derive the first order partial differential equation in $u(x,y)$ if its general solution is given by

$$u(x,y) = F(2x-3y),$$

where F is an arbitrary function. Hence, find the solution of the resulting equation if $u(x,0) = x^2$.

(4 marks)

(b) Use the method of characteristics to solve the initial-value problem

$$u_x - xu_y + u = e^{-x}, \qquad u(x,0) = 0.$$

(8 marks)

(c) Solve the non-linear equation

$$u_t + uu_x = 0, \qquad u(x,0) = f(x),$$

where $f(x) = \begin{cases} x & , \quad 0 \le x < 1, \\ 2-x & , \quad 1 \le x \le 2, \\ 0 & , \quad x > 2. \end{cases}$

Hence, find and sketch the solution at $t = \dfrac{1}{3}$.

(8 marks)

Q2 (a) Consider the periodic function

$$f(x) = \begin{cases} 0, & -\pi < x < 0, \\ x, & 0 < x < \pi, \end{cases}$$

and $f(x) = f(x+2\pi)$.

(i) Determine whether the function is even, odd or neither. State your reasons.

(3 marks)

(ii) Compute its Fourier series.

(11 marks)

(b) Find the half-range Sine series of

$$f(x) = 1 - x, \qquad 0 < x < 1.$$

(6 marks)

Q3 Given the wave equation

$$u_{tt} = 4u_{xx}, \quad 0 < x < \pi, \quad t > 0,$$

that satisfies the conditions

$$u(0,t) = 0, \quad u(\pi,t) = 0, \quad t > 0$$
$$u(x,0) = 0, \quad u_t(x,0) = V, \quad 0 < x < \pi.$$

(a) State the initial and boundary conditions of the wave problem above.

(2 marks)

(b) By assuming $u(x,t) = X(x)T(t)$, show that the wave equation above can be reduced to

$$X'' - pX = 0 \text{ and } T'' - 4pT = 0,$$

where p is constant.

(4 marks)

(c) By considering $p = 0$, $p = \lambda^2$ for $p > 0$ and $p = -\lambda^2$ for $p < 0$, and by applying $u(0,t) = 0$ and $u(\pi,t) = 0$, show that

$$u(x,t) = \sum_{n=1}^{\infty} (\sin nx)(P_n \cos 2nt + Q_n \sin 2nt),$$

where P and Q are constants.

(7 marks)

(d) By applying $u(x,0) = 0$ and $u_t(x,0) = V$, find P_n and Q_n. Hence, write the solution to the above wave problem.

(7 marks)

Q4 (a) Show that $u(x,t) = 3e^{-28t}\sin 2x - 6e^{-175t}\sin 5x$ is a solution of heat equation $u_t = 7u_{xx}$.

(4 marks)

(b) The mathematical model for the heat diffusion in a uniform wire without internal sources whose ends are kept at the constant temperature $0^\circ C$ with initial temperature distribution $100^\circ C$ is given as

$$u_t = 4u_{xx}, \qquad 0 < x < 1, \quad t > 0.$$

(i) State the boundary and initial conditions for the above heat problem.

(2 marks)

(ii) Solve this problem using the method of separation of variables.

(14 marks)

Q5 The temperature distribution $u(r,\theta)$ in a circular metal disc of radius 2 that has its top and bottom insulated is described by the equation

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = u_t, \quad 0 < r < 2, \quad 0 \le \theta \le 2\pi, \quad t > 0.$$

(a) State the condition so that the temperature distribution $u(r,\theta)$ satisfies the Laplace equation.

(2 marks)

(b) Show that the general solution of the Laplace equation

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0,$$

is $u(r,\theta) = (A\ln r + B)(C\theta + D) + (Er^k + Fr^{-k})(R\cos k\theta + S\sin k\theta)$,

where *A, B, C, D, E, F, R* and *S* are constants.

(8 marks)

(c) State the condition required so that the solution of the Laplace equation in 5(b) can be written as

$$u(r,\theta)=\frac{P_0}{2}+r^k(P\cos k\theta+Q\sin k\theta),$$

where P and Q are constants.

(3 marks)

(d) If $u(2,\theta)=5$, find the particular solution of the general solution of Laplace problem in 5(c).

(7 marks)

- END OF QUESTION -

FINAL EXAMINATION

SEMESTER / SESSION: SEM I / 2014/2015 COURSE : 3BWA
SUBJECT : PARTIAL DIFFERENTIAL EQUATION CODE : BWA30303

Formulae

Fourier Series: $f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}\left\{a_n\cos\left(\frac{n\pi x}{\ell}\right)+b_n\sin\left(\frac{n\pi x}{\ell}\right)\right\},$

where $a_0=\frac{1}{\ell}\int_{-\ell}^{\ell}f(x)\,dx,$

$$a_n=\frac{1}{\ell}\int_{-\ell}^{\ell}f(x)\cos\left(\frac{n\pi x}{\ell}\right)dx,\quad n=1,2,3,...,$$

$$b_n=\frac{1}{\ell}\int_{-\ell}^{\ell}f(x)\sin\left(\frac{n\pi x}{\ell}\right)dx,\quad n=1,2,3,...$$

Half Range Cosine Series: $f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}a_n\cos\left(\frac{n\pi x}{\ell}\right),$

where $a_0=\frac{2}{\ell}\int_{0}^{\ell}f(x)\,dx,$

$$a_n=\frac{2}{\ell}\int_{0}^{\ell}f(x)\cos\left(\frac{n\pi x}{\ell}\right)dx,\quad n=1,2,3,...$$

Half Range Sine Series: $f(x)=\sum_{n=1}^{\infty}b_n\sin\left(\frac{n\pi x}{\ell}\right),$

where $b_n=\frac{2}{\ell}\int_{0}^{\ell}f(x)\sin\left(\frac{n\pi x}{\ell}\right)dx,\quad n=1,2,3,...$