CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2012/2013

COURSE NAME	:	CALCULUS II
COURSE CODE	:	BWA 10503
PROGRAMME	:	1 BWA / 1 BWQ
EXAMINATION DATE	:	JUNE 2013
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **FIVE (5)** QUESTIONS.

THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

Q1 (a) Let $f(x) = \sin 2x$.

(i) Show that the Taylor series for $f(x)$ about $x = 0$ is

$$\sum_{n=0}^{\infty} (-1)^n \frac{2^{2n+1} x^{2n+1}}{(2n+1)!}.$$

(ii) Determine whether the series in (i) absolutely converges, conditionally converges or diverges.

(iii) Hence, find the radius and interval of convergence of the given series.

(11 marks)

(b) Given that the Maclaurin series of $\cos x$ is

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

Hence, find the first three terms of the series for

(i) $\dfrac{d}{dx} \cos x^2$.

(ii) $\int \cos x^2 \, dx$.

(9 marks)

Q2 Let $f(x, y) = x^2 + y^2 - 4y + 5$.

(a) Determine the domain and range of $f(x, y)$.

(4 marks)

(b) Sketch the contour map of the function using level curves $c = 2$, $c = 5$ and $c = 10$.

(5 marks)

(c) Hence, sketch the 3D graph for $f(x, y)$.

(2 marks)

(d) Use 2^{nd} partial derivative test to obtain the relative extrema of $f(x, y)$.

(7 marks)

(e) Does $f(x, y)$ has a saddle point? Give your reason.

(2 marks)

Q3 (a) Let

$$f(x,y)=\begin{cases}\dfrac{x+y-4}{\sqrt{x+y}-2}, & (x,y)\neq(1,\ 3),\\ 6, & (x,y)=(1,\ 3).\end{cases}$$

(i) Evaluate $\lim\limits_{(x,y)\to(1,\ 3)} f(x,y)$ if it exists.

(ii) Hence, determine whether the function $f(x, y)$ continuous or not at (1, 3).

(8 marks)

(b) Given $z=\dfrac{xy}{x-y}$, $x=v+uv$ and $y=uv$.

(i) Verify that the function z is a solution of the differential equation

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=z.$$

(ii) Show that $\dfrac{\partial z}{\partial u}=v(1+2u)$.

(12 marks)

Q4 (a) Evaluate $\displaystyle\int_0^{\ln 3}\int_0^{\ln 2} e^{x+y}\,dy\,dx$.

(4 marks)

(b) Compute $\displaystyle\int_0^{\pi/2}\int_0^{2\cos\theta} r^2\,dr\,d\theta$.

(6 marks)

(c) Let

$$\iiint_G z^2\sqrt{x^2+y^2+z^2}\;dV$$

where G is the region given by $-5\leq x\leq 5$, $-\sqrt{25-x^2}\leq y\leq\sqrt{25-x^2}$ and $0\leq z\leq\sqrt{25-x^2-y^2}$.

(i) Sketch the region G.

(ii) By changing to spherical coordinates, evaluate the given integration.

(10 marks)

Q5 Let G be a solid bounded above by the paraboloid $z = 2 + x^2 + y^2$ and below by the plane $z = 11$ in the first octant.

(a) Evaluate the volume of the solid.

(6 marks)

(b) Calculate the surface area of the solid that lies under the plane $z = 11$.

(7 marks)

(c) Find the mass of the solid if the solid's density at (x, y, z) is

$$\delta(x, y) = \frac{2}{\sqrt{x^2 + y^2}}.$$

(7 marks)

- END OF QUESTION -

FINAL EXAMINATION

SEMESTER / SESSION : SEM II / 2012/2013
COURSE : CALCULUS II

PROGRAMME : 1 BWA / 1 BWQ
COURSE CODE : BWA 10503

Formulae

TAYLOR AND MACLAURIN SERIES

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

PARTIAL DIFFERENTIATION

1st Partial Derivative Test

$$f_x(a,b) = 0 \text{ and } f_y(a,b) = 0$$

2nd Partial Derivative Test

$$d = f_{xx}(a,b) f_{yy}(a,b) - [f_{xy}(a,b)]^2$$

Chain Rule

Let $w = f(x, y, z)$ and $x = g(u, v)$, $y = h(u, v)$ and , $z = k(u, v)$, then

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial u} \text{ and } \frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial v}$$

MULTIPLE INTEGRATIONS

Polar coordinates

$x = r\cos\theta$, $y = r\sin\theta$ and $r^2 = x^2 + y^2$

$$\iint_R f(x,y)\, dA = \int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2} f(r,\theta)\, r\, dr\, d\theta$$

Cylindrical coordinates

$x = r\cos\theta$, $y = r\sin\theta$, $z = z$ and $r^2 = x^2 + y^2$

$$\iiint_G f(x,y,z)\, dV = \int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\int_{z_1}^{z_2} f(z,r,\theta)\, r\, dz\, dr\, d\theta$$

Spherical coordinates

$x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $z = \rho\cos\phi$, $x^2 + y^2 + z^2 = \rho^2$, $0 \le \phi \le \pi$ and $0 \le \theta \le 2\pi$

$$\iiint_G f(x,y,z)\, dV = \int_{\theta_1}^{\theta_2}\int_{\phi_1}^{\phi_2}\int_{\rho_1}^{\rho_2} f(\rho,\phi,\theta)\, \rho^2 \sin\phi \; d\rho\, d\phi\, d\theta$$

Surface Area

$$A(S) = \iint_R \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\; dA$$

Mass

$$m = \iint_R \delta(x,y)\, dA$$

$$m = \iiint_G \delta(x,y,z)\, dV$$