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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2014/2015

COURSE NAME	:	MATHEMATICS FOR ENGINEERING TECHNOLOGY III
COURSE CODE	:	BWM 22403
PROGRAMME	:	1 BND / 1 BNE / 1 BNF / 1 BNG / 1 BNL / 1 BNM / 2 BND / 2 BNE / 2 BNF / 2 BNG / 2 BNH / 2 BNL / 2 BNM
EXAMINATION DATE	:	DECEMBER 2014 / JANUARY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER **ALL** QUESTIONS 2. ALL CALCULATIONS MUST BE IN **3 DECIMAL** PLACES

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

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Q1 (a) A violin string of unit length is stretched out horizontally with both ends fixed satisfies the wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \quad t > 0$$

with the initial conditions, $u(x,0) = \sin(4\pi x)$, $u_t(x,0) = 0$ for $0 \le x \le 1$. By taking $h = \Delta x = 0.2$ and $k = \Delta t = 0.1$, find the displacement of the violin string up to level 2 only by using the explicit finite difference method.

(10 marks)

(b) Given a matrix

$$A = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 4 \end{bmatrix}.$$

(i) Find the inverse of matrix A by calculator.

(1 mark)

(ii) Find the smallest (in absolute) eigenvalue and its corresponding eigenvector by inverse power method.

Let $v^{(0)} = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}^T$. Do your computation in 3 decimal places.

(9 marks)

Q2 (a) A current i is travelling through a single turn loop of radius 1 m. A four-turn search coil of effective area 0.03 m^2 is placed inside the loop. The magnetic flux in weber (Wb) linking the search coil is given by

$$\phi = \mu_0 \frac{iA}{2r},$$

where r is the radius of the current carrying loop, A is the area of the search coil and μ_0 is the permeability of free space.

Find the value of electromotive force (emf), ε induced in the search coil at time, $t = 3$ sec and $t = 5.5$ sec by using 3-point central difference formula with $h = 0.01$ if ε is given by

$$\varepsilon = -N\frac{d\phi}{dt},$$

where N is the number of turns in the search coil, the current is $i = 20\sin(20\pi t) + 50\sin(30\pi t)$ and $\mu_o = 4\pi \times 10^{-7}$ Hm^{-1}.

(10 marks)

(b) The area between two curves is given by formula $\int_a^b [f(x)-g(x)]\,dx$ where $f(x)$ is upper boundary and $g(x)$ is lower boundary. Calculate the area bounded by the graph of $y=3-x$ and $y=x^2-9$ between $x=0$ and $x=3$ by using

(i) 2-point Gauss-quadrature.

(8 marks)

(ii) 3-point Gauss-quadrature.

(2 marks)

Q3 (a) Let $f(x)=x^4-18x^2+45$.

(i) Verify that $f(x)$ has a root on the interval $(1,2)$.

(1 mark)

(ii) Find the root of $f(x)$ by using secant method and iterate until $|f(x_i)|<\varepsilon=0.005$.

(5 marks)

(iii) Given that the exact value of the root is $x=\sqrt{3}$. Compute the absolute error in the approximation in **Q3**(a)(ii).

(2 marks)

(b) Solve the system of linear equations below by using Thomas algorithm method. Rearrange the rows if necessary.

$$\begin{pmatrix} 0 & 0 & 3 & 4 \\ 2 & 9 & 1 & 0 \\ 0 & 1 & 9 & 4 \\ 2 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}=\begin{pmatrix} 8 \\ 6 \\ 8 \\ 9 \end{pmatrix}$$

(12 marks)

Q4 (a) The upward velocity of a rocket, measured at 3 different times, is shown in the table below.

Table 1: Upward velocity of a rocket

Time , t (seconds)	Velocity, v (meters/second)
5	106.8
8	177.2
12	279.2

The velocity over the time interval $5 \leq t \leq 12$ is approximated by a quadratic expression as

$$v(t) = a_1 t^2 + a_2 t + a_3 .$$

Find the values of a_1, a_2 and a_3 by using Gauss – Elimination method.

(10 marks)

(b) Torque-speed data for an electric motor is given in the first two columns of the table below. Find the equation of the Newton divided–difference interpolating polynomial that passes through each data point and use it to estimate the torque at 1800 rpm.

Table 2: Torque-speed for an electric motor

Speed ω (rpm $\times$ 1000)	0.5	1.0	1.5	2.0	2.5
Torque, T_i (ft-lb)	31	28	24	14	2

(10 marks)

Q5 (a) Find the local extreme and saddle point (if exists) for the function

$$f(x, y) = 2x^2 - y^3 - 2xy + 4 .$$

(10 marks)

(b) Find the volume of the solid within the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $y + z = 2$ by using cylindrical coordinates.

(10 marks)

- END OF QUESTION -

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Formulas

Partial differential equations

Wave equation: Finite-difference method:

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \quad \Leftrightarrow \quad \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

$$\frac{\partial u(x,0)}{\partial t} = \frac{u_{i,j+1} - u_{i,j-1}}{2k} = g(x_i)$$

Eigenvalue

Power Method: $v^{(k+1)} = \frac{1}{m_{k+1}} A v^{(k)}, \qquad k = 0,1,2,.....$

Inverse Power Method: $\lambda_{\text{smallest}} = \frac{1}{\lambda_{\text{Shifted}}}$

Numerical differentiation and integration

Differentiation:

First derivatives:

2-point forward difference: $f'(x) \approx \frac{f(x+h) - f(x)}{h}$

2-point backward difference: $f'(x) \approx \frac{f(x) - f(x-h)}{h}$

3-point forward difference: $f'(x) \approx \frac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h}$

3-point backward difference: $f'(x) \approx \frac{3f(x) - 4f(x-h) + f(x-2h)}{2h}$

3-point central difference: $f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$

5-point difference: $f'(x) \approx \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}$

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Integration:

Gauss quadrature: For $\int_a^b f(x)dx$, $\quad x = \dfrac{(b-a)t + (b+a)}{2}$

2-points: $\int_{-1}^{1} f(x)dx \approx g\left(-\frac{1}{\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right)$

3-points: $\int_{-1}^{1} f(x)dx \approx \frac{5}{9} g\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} g(0) + g\left(\sqrt{\frac{3}{5}}\right)$

Nonlinear equations

Secant method : $x_{i+2} = \dfrac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}$, $i = 0,1,2,\ldots$

System of linear equations

Thomas Algorithm:

i	**1**	**2**	**…**	***n***
d_i				
e_i				
c_i				
b_i				
$\alpha_1 = d_1$ $\alpha_i = d_i - c_i\beta_{i-1}$				
$\beta_i = \dfrac{e_i}{\alpha_i}$				
$y_1 = \dfrac{b_1}{\alpha_1}$ $y_i = \dfrac{b_i - c_i y_{i-1}}{\alpha_i}$				
$x_n = y_n$ $x_i = y_i - \beta_i x_{i+1}$				

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Interpolation

Newton divided difference:

$$P_n(x) = f_0^{[0]} + f_0^{[1]}(x - x_0) + f_0^{[2]}(x - x_0)(x - x_1) + \ldots + f_0^{[n]}(x - x_0)(x - x_1)\ldots(x - x_{n-1})$$

Local extreme value

$$G(x,y) = f_{xx}(x,y) \times f_{yy}(x,y) - \left[f_{xy}(x,y)\right]^2$$

$G(a,b) > 0$ and $f_{xx}(a,b) < 0$ $\Rightarrow$ local maximum value

$G(a,b) > 0$ and $f_{xx}(a,b) > 0$ $\Rightarrow$ local minimum value

$G(a,b) < 0$ $\Rightarrow$ saddle point

$G(a,b) = 0$ $\Rightarrow$ test is inconclusive

Cylindrical coordinates:

$$x = r\cos\theta, \quad y = r\sin\theta, \quad z = z, \qquad x^2 + y^2 = r^2, \quad 0 \le \theta \le 2\pi$$

$$V = \iiint_G dV = \iiint_G dz\, r\, dr\, d\theta$$