UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2014/2015

COURSE NAME	:	GRAPH THEORY
COURSE CODE	:	BWA 20203
PROGRAMME	:	2 BWA
EXAMINATION DATE	:	DECEMBER 2014 / JANUARY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **FIVE** QUESTIONS **ONLY**

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

Q1 (a) Check if the graph as in **Figure Q1(a)** fulfill the condition mentioned in Dirac's theorem and Ore's theorem.

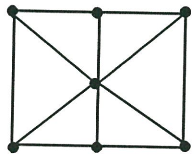


Figure Q1 (a)

Hence, determine whether the graph has a Hamiltonian cycle. If yes, find the cycle. If it does not, give an argument to show that why no such cycle exist.

(5 marks)

(b) Use Prim's algorithm and suitable shortcut to find the upper bound for the solution to the travelling salesman problem for the network as in **Figure Q1 (b).** Then, find the lower bound for the problem.

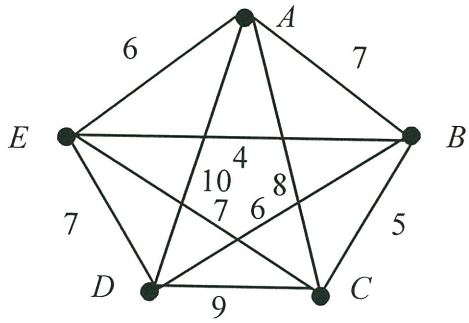


Figure Q1 (b)

(11 marks)

(c) By writing in tabular form, find the length of a shortest path between s and t in the given weighted graph as in **Figure Q1 (c).**

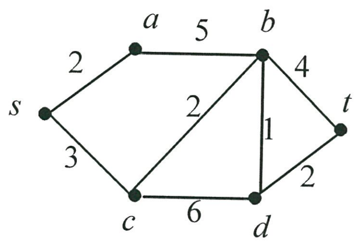


Figure Q1 (c)

(4 marks)

Q2 (a) How many continuous penstrokes are required to draw the following diagram in **Figure Q2(a)** without covering any part twice?

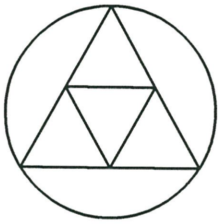


Figure Q2 (a)

(2 marks)

(b) By using matrix tree theorem , show that K_4 has 16 spanning trees.

(4 marks)

(c) (i) Find the Prüfer sequence for the labelled tree as in **Figure Q2(c)(i).**

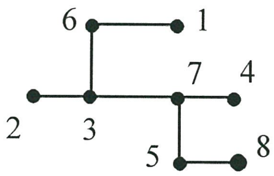


Figure Q2 (c)(i)

(4 marks)

(ii) Reconstruct labelled tree from the Prüfer sequence (3,4,4,5,1) as in the pattern of **Figure Q2(c)(ii).**

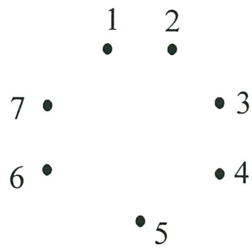


Figure Q2 (c)(i)

(4 marks)

(d) Show that there is no solution for the following four cubes problem as in **Figure Q2(c)** .

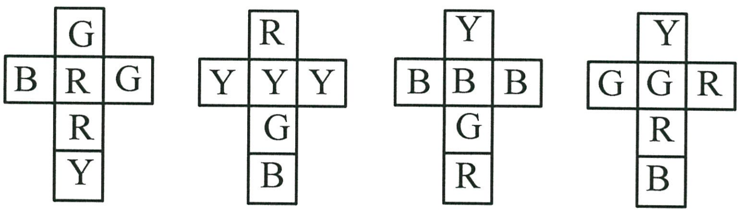
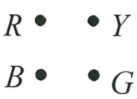


Figure Q2 (c)

Hint : The letters R, Y, B and G stand for the colors red, yellow, blue and green. Please draw your graph as



(6 marks)

Q3 (a) Verify the Euler formula (i.e. Euler polyhedral equation) for plane graph as shown in **Figure Q3 (a)** .

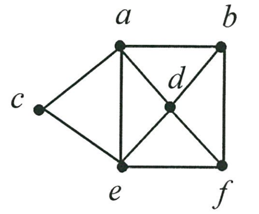


Figure Q3 (a)

(2 marks)

(b) Show that the graphs in **Figure Q3 (a)** and **Figure Q3 (b)** are isomorphic, but their geometric dual are non isomorphic.

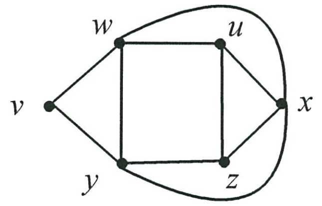


Figure Q3 (b)

(6 marks)

(c) State the Kuratowski's theorem. Hence, by using the Kuratowski's theorem, proof that K_6 is not a planar graph.

(4 marks)

(d) By using edge-face inequality, check if

(i) K_5 can be embedded on disk,

(ii) K_4 can be embedded on torus,

(iii) $K_{3,3}$ can be embedded on sphere.

(Hint: Euler characteristic for torus is 0, disk is 1 and sphere is 2. If the test is inconclusive, write "the test is inconclusive").

(8 marks)

Q4 (a) State the Brook's theorem. What does Brook's theorem tell you about the chromatic number of the graph in **Figure Q4 (a)** ? Hence, determine the chromatic number and chromatic index for the graph in **Figure Q4 (a)**.

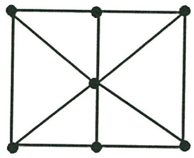


Figure Q4 (a)

(5 marks)

(b) Find the chromatic polynomial $P_G(k)$ for the graph in **Figure Q4 (b)** by

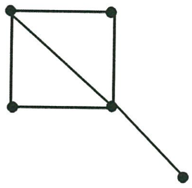


Figure Q4 (b)

(i) adding edge(s) and contracting the edge(s),
(ii) deleting edge(s) and contracting the edge(s),

Show that both ways give the same result.
(Hint: Chromatic polynomial for $P_{K_n}(k) = k(k-1)(k-2)\cdots(k-(n-1))$, $P_{T_n}(k) = k(k-1)^{n-1}$, $P_{C_n}(k) = (k-1)^n + (-1)^n(k-1))$).

(9 marks)

(c) Given that a simple graph, G, with chromatic polynomial
$P_G(k) = k^6 - 15k^5 + 85k^4 - 225k^3 + 274k^2 - 120k$

(i) How many edges does G have?
(ii) By using the above chromatic polynomial , is G a bipartite graph?
(iii) How many ways to colour the graph, G with 5 colours?
(iv) Based on (iii), is the graph a planar graph? Why?

(6 marks)

Q5 (a) Check if the digraph in **Figure Q5 (a)** is strongly connected or not. Explain your answer. Then, check also the underlying graph for the digraph in **Figure Q5 (a)** is orientable or not, why?

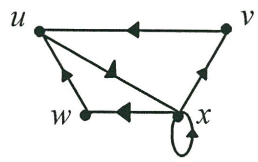


Figure Q5 (a)

(4 marks)

(b) Based on the following adjacency matrix, draw the digraph. Check if the digraph is Eulerian digraph? Explain your answer.

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

(6 marks)

(c) For the digraph in **Figure Q5(c),**

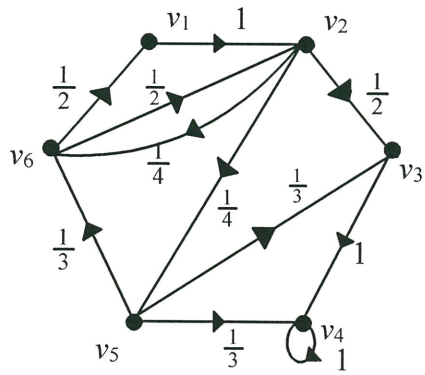


Figure Q5 (c)

(i) Write the transition matrix.

(2 marks)

(ii) Complete **Table Q5(c)**.

Table Q5(c)

State	Persistent/ Transient	Absorbing state(YES/NO)	Periodic/ Aperiodic	Ergodic state (YES/NO)
E_1				
E_2				
E_3				
E_4				
E_5				
E_6				

Remark : Write down the period if it is periodic.

(8 marks)

Q6 (a) Explain the meaning for term rank of (0-1)-matrix by using König-Egerváry Theorem. Hence, explain how to determine that transversal exist.

(3 marks)

(b) (i) By using (0-1)-matrix, determine whether the following family of subsets of $E = \{1,2,3,4,5,6\}$ have transversal or not?

$$\{1,2\},\ \{2,3,4\},\ \{2,5,6\},\ \{1,3,4\},\ \{2\},\ \{1,4\}$$

(4 marks)

(ii) By using (0-1)-matrix, determine the following marriage problem as in **Table Q6(b) (ii)** has complete matching or not. If yes, write down the perfect matching.

Table Q6(b)(ii)

Boy	Girls known by boy
b_1	g_2, g_4
b_2	g_1
b_3	g_1, g_3, g_5
b_4	g_2, g_4, g_6
b_5	g_2, g_6

(5 marks)

(c) For the graph in **Figure Q6(c)**

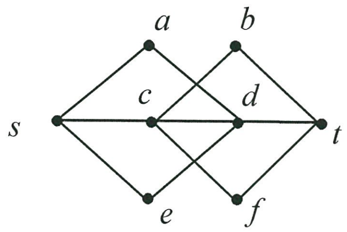


Figure Q6(c)

(i) Write down all edge-disjoint st - path.
(ii) Write down a set of all edges separating s from t.
Hence, verify the edge-form of Menger's theorem for the graph.

(4 marks)

(d) For the graph in **Figure Q6(d)**

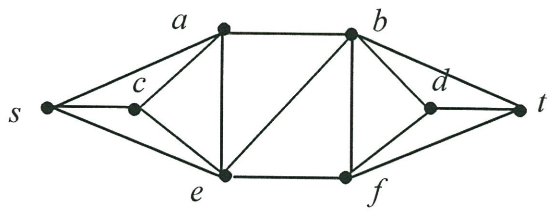


Figure Q6(d)

(i) Write down all vertex-disjoint st - path.
(ii) Write down a set of all vertices separating s from t.
Then, verify the vertex-form of Menger's theorem for the graph.

(4 marks)

- END OF QUESTION -