UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2013/2014

COURSE NAME	:	ENGINEERING MATHEMATICS I
COURSE CODE	:	BWM10103/BSM1913
PROGRAMME	:	1 BEB, BEC, BEE, BEH, BEJ, BEV, BDD, BFF
EXAMINATION DATE	:	DECEMBER 2013/JANUARY 2014
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS IN SECTION (A). B) ANSWER **THREE (3)** QUESTIONS ONLY IN SECTION (B).

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

SECTION A

Q1 (a) Differentiate the following functions with respect to x.

(i) $f(x)=\dfrac{\tanh x}{x^2}$

(3 marks)

(ii) $f(x)=\sqrt{x^2+1}\sinh^{-1}x$

(3 marks)

(b) Evaluate

$$\int \frac{x}{\sqrt{3-2x-x^2}}\,dx \,.$$

(7 marks)

(c) Find the area of the surface that is generated by revolving the portion of the curve

$$f(x)=\sqrt{4-x^2}, \ -1\le x\le 1$$

360^0 about the x-axis.

(7 marks)

Q2 (a) Find the first four nonzero terms of the Maclaurin series for the function

$$f(x)=\frac{1}{(1+2x)^2}.$$

(6 marks)

(b) Given that $\cos x = 1-\dfrac{x^2}{2!}+\dfrac{x^4}{4!}-\dfrac{x^6}{6!}+\ldots$.

(i) Derive the power series for $\sin x$.

(2 marks)

(ii) Find the first three nonzero terms of the power series for $\dfrac{\sin x}{\sqrt{x}}$.

(1 mark)

(iii) Hence, evaluate

$$\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{\sin x}{\sqrt{x}}\,dx$$

by using the series expansion in part (b)(ii).

(3 marks)

(c) (i) Determine whether the series $\sum_{n=1}^{\infty} \frac{n^2}{e^n}$ converges or diverges by using the ratio test.

(3 marks)

(ii) Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converges absolutely, converges conditionally, or diverges.

(5 marks)

SECTION B

Q3 (a) Give the conditions that a function *f* must satisfy so that it is continuous at $x = a$.

(2 marks)

(b) Find the constants *a* and *b* so that the following function is continuous for all *x*.

$$f(x) = \begin{cases} \frac{\sin ax}{x}, x < 0 \\ 5, x = 0 \\ x + b, x > 0 \end{cases}.$$

(7 marks)

(c) Use the L'Hôpital's rule to determine the following limits:

(i) $\lim_{x \to 1} \frac{\ln x}{x^2 - x}$,

(2 marks)

(ii) $\lim_{x \to 0} \frac{2\cos x - 2 + x^2}{3x^4}$,

(5 marks)

(iii) $\lim_{x \to 0} \frac{e^x + e^{-x} - 2}{1 - \cos 2x}$.

(4 marks)

Q4 (a) Find $\frac{dy}{dx}$ if

(i) $y = (4x-1)e^{-2x^2}$,

(3 marks)

(ii) $y = e^{x\tan x}$.

(4 marks)

(b) Let

$$f(x) = x^3 - 5x^2 + 7x - 3.$$

(i) Find all critical numbers. Then use the Second Derivative Test to determine the properties of all the local extreme points.

(6 marks)

(ii) Locate the point of inflection.

(4 marks)

(iii) Sketch the graph of f base on the information obtained in part (b)(i) and (b)(ii).

(3 marks)

Q5 (a) With a suitable substitution, find

$$\int \frac{x}{\sqrt{x+4}}\,dx.$$

(5 marks)

(b) Integrate $f(x)$ with respect to x by using the tabular method.

$$\int x^5 \sin 3x\,dx.$$

(6 marks)

(c) Show that

$$\frac{6x+7}{(x+2)^2} \equiv \frac{6}{x+2} - \frac{5}{(x+2)^2}.$$

Then evaluate

$$\int_{-1}^{0} \frac{6x+7}{(x+2)^2}\,dx.$$

(9 marks)

Q6 (a) Find $\frac{dy}{dx}$ if

(i) $y = 3\tan^{-1}\left(\frac{x-1}{2}\right)$,

(3 marks)

(ii) $y = \sinh^{-1}\left(\frac{1}{3}x\right)$.

(3 marks)

(b) Evaluate

$$\int \sinh^3 x \cosh^2 x \, dx.$$

(Hint: use the identity $\cosh^2 x - \sinh^2 x = 1$).

(6 marks)

(c) Find the circumference of a circle of radius a from the parametric equations

$$x = a\cos t,\ y = a\sin t,$$

where $0 \le t \le 2\pi$. Then show that the radius of curvature at any point on the circumference of a circle is equal to its radius.

(8 marks)

- END OF QUESTION -

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INTEGRATION

Indefinite Integrals	Integration of Inverse Functions
$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C$
$\int \frac{1}{x} dx = \ln\lvert x \rvert + C$	$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + C$
$\int \cos x \, dx = \sin x + C$	$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \frac{-1}{a^2 + x^2} dx = \frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right) + C$
$\int \sec^2 x \, dx = \tan x + C$	$\int \frac{1}{\lvert x \rvert \sqrt{x^2 - a^2}} dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$
$\int \csc^2 x \, dx = -\cot x + C$	$\int \frac{-1}{\lvert x \rvert \sqrt{x^2 - a^2}} dx = \frac{1}{a}\csc^{-1}\left(\frac{x}{a}\right) + C$
$\int \sec x \tan x \, dx = \sec x + C$	$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \sinh^{-1}\left(\frac{x}{a}\right) + C$
$\int \csc x \cot x \, dx = -\csc x + C$	$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \cosh^{-1}\left(\frac{x}{a}\right) + C$
$\int e^x dx = e^x + C$	$\int \frac{-1}{\lvert x \rvert \sqrt{a^2 - x^2}} dx = \frac{1}{a}\operatorname{sech}^{-1}\left\lvert\frac{x}{a}\right\rvert + C$
$\int \cosh x \, dx = \sinh x + C$	$\int \frac{-1}{\lvert x \rvert \sqrt{a^2 + x^2}} dx = \frac{1}{a}\operatorname{csch}^{-1}\left\lvert\frac{x}{a}\right\rvert + C$
$\int \sinh x \, dx = \cosh x + C$	$\int \frac{1}{a^2 - x^2} dx = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C, & \lvert x \rvert < a \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C, & \lvert x \rvert > a \end{cases}$
$\int \operatorname{sech}^2 x \, dx = \tanh x + C$	
$\int \operatorname{csch}^2 x \, dx = -\coth x + C$	
$\int \operatorname{sech} x \tanh x \, dx = -\operatorname{sech} x + C$	
$\int \operatorname{csch} x \coth x \, dx = -\operatorname{csch} x + C$	

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CURVATURE, ARC LENGTH AND SURFACE AREA OF REVOLUTION

$$\kappa = \frac{\left|\frac{d^2y}{dx^2}\right|}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}} = \frac{|\dot{x}\ddot{y}-\dot{y}\ddot{x}|}{\left[\dot{x}^2+\dot{y}^2\right]^{\frac{3}{2}}},$$

$$S = 2\pi\int_a^b f(x)\sqrt{1+\left(\frac{d}{dx}[f(x)]\right)^2}\,dx = 2\pi\int_c^d g(y)\sqrt{1+\left(\frac{d}{dy}[g(y)]\right)^2}\,dy$$

$$L = \int_{t_1}^{t_2}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt = \int_a^b\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_c^d\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC FUNCTIONS

Trigonometric Functions	Hyperbolic Functions
$\cos^2 x+\sin^2 x=1$	$\sinh x=\frac{e^x-e^{-x}}{2}$
$\sin 2x=2\sin x\cos x$	$\cosh x=\frac{e^x+e^{-x}}{2}$
$\cos 2x=\cos^2 x-\sin^2 x$	$\cosh^2 x-\sinh^2 x=1$
$=2\cos^2 x-1$	$\sinh 2x=2\sinh x\cosh x$
$=1-2\sin^2 x$	$\cosh 2x=\cosh^2 x+\sinh^2 x$
$1+\tan^2 x=\sec^2 x$	$=2\cosh^2 x-1$
$1+\cot^2 x=\csc^2 x$	$=1+2\sinh^2 x$
$\tan 2x=\frac{2\tan x}{1-\tan^2 x}$	$1-\tanh^2 x=\text{sech}^2 x$
$\tan(x\pm y)=\frac{\tan x\pm\tan y}{1\mp\tan x\tan y}$	$\coth^2 x-1=\text{csch}^2 x$
$\sin(x\pm y)=\sin x\cos y\pm\sin y\cos x$	$\tanh 2x=\frac{2\tanh x}{1+\tanh^2 x}$
$\cos(x\pm y)=\cos x\cos y\mp\sin x\sin y$	$\tanh(x\pm y)=\frac{\tanh x\pm\tanh y}{1\pm\tanh x\tanh y}$
$\sin ax\cos bx=\frac{1}{2}[\sin(a+b)x+\sin(a-b)x]$	$\sinh(x\pm y)=\sinh x\cosh y\pm\sinh y\cosh x$
$\sin ax\sin bx=\frac{1}{2}[-\cos(a+b)x+\cos(a-b)x]$	$\cosh(x\pm y)=\cosh x\cosh y\pm\sinh x\sinh y$
$\cos ax\cos bx=\frac{1}{2}[\cos(a+b)x+\cos(a-b)x]$	

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SUBSTITUTIONS

Expression	*Trigonometric*	*Hyperbolic*
$\sqrt{x^2+k^2}$	$x = k\tan\theta$	$x = k\sinh\theta$
$\sqrt{x^2-k^2}$	$x = k\sec\theta$	$x = k\cosh\theta$
$\sqrt{k^2-x^2}$	$x = k\sin\theta$	$x = k\tanh\theta$

$t = \tan\frac{1}{2}x$

$\sin x = \dfrac{2t}{1+t^2}$ $\quad \cos x = \dfrac{1-t^2}{1+t^2}$

$\tan x = \dfrac{2t}{1-t^2}$ $\quad dx = \dfrac{2dt}{1+t^2}$

$t = \tan x$

$\sin 2x = \dfrac{2t}{1+t^2}$ $\quad \cos 2x = \dfrac{1-t^2}{1+t^2}$

$\tan 2x = \dfrac{2t}{1-t^2}$ $\quad dx = \dfrac{dt}{1+t^2}$

TAYLOR AND MACLAURIN SERIES

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots$$