Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESI 2013/2014

COURSE NAME	:	STATISTICAL INFERENCE
COURSE CODE	:	BWB 20503
PROGRAMME	:	2 BWQ
EXAMINATION DATE	:	DECEMBER 2013 /JANUARY 2014
DURATION	:	2 HOURS 30 MINUTES
INSTRUCTION	:	ANSWER ALL QUESTIONS

THIS EXAMINATION PAPER CONSISTS OF **FOUR (4)** PAGES

Q1 (a) In each of the following find the probability density function of Y. Show that integrates to 1.

(i) $Y = X^3$ and $f_X(x) = 15x^5(1-x)$, $0 < x < 1$.

(4 marks)

(ii) $Y = 4X + 1$ and $f_X(x) = ½e^{-1/2\, x}$, $0 < x < \infty$

(4 marks)

(b) Let θ be a fixed positive constant, and define the function $f(x)$ by

$$f(x) = \begin{cases} \frac{1}{2}\theta e^{-\theta x} & if \quad x \geq 0 \\ \frac{1}{2}\theta e^{\theta x} & if \quad x < 0 \end{cases}$$

(i) Verify that $f(x)$ is a probability density function.

(4 marks)

(ii) If X is a random variable with pdf given by $f(x)$, find $P(X < t)$ for all t. Evaluate all integrals.

(7 marks)

(iii) Find $P(|X| < t)$ for all t. Evaluate all integrals.

(6 marks)

Q2 Let $X_1, \ldots, X_n$ be independent and identically distributed random variables with probability density function

$$f_X(x|\gamma) = 3\gamma x^2 e^{-\gamma x^3} \quad x > 0, \gamma > 0.$$

(a) Find sufficient statistics for γ and state the result that you are using.

(3 marks)

(b) Derive the score statistic, $U(X)$.

(3 marks)

(c) Derive the maximum likelihood estimator, $\hat{\gamma}$, of γ.

(3 marks)

(d) Drive Fisher information for γ.

(3 marks)

(e) Provide the maximum likelihood estimator, $\hat{g}(X)$, of $\frac{1}{\gamma}$.

(4 marks)

(f) Show that , $\hat{g}(X)$ is unbiased estimator of $\frac{1}{\gamma}$ that attains the Cramer-Rao Lower Bound.

(5 marks)

(g) Find Var[$\hat{g}(X)$].

(4 marks)

Q3 (a) Let $X_1, \ldots, X_n$ be independent Uniform random variables with probability density function

$$f(x_i|\theta) = \begin{cases} \frac{1}{\theta} & 0 \leq x_i \leq \theta \\ 0 & otherwise \end{cases}$$

(i) State the likelihood function $f(x \ \ |\theta)$.

(3 marks)

(iii) State Neyman's Factorisation Theorem.

(2 marks)

(b) Let $X_1, X_2, \ldots, X_n$ random variables from $N(\mu, \sigma^2)$. Suppose that σ is known, so that μ is the parameter, and consider the random variable $T_n(\mu) = \frac{(\bar{X}-\mu)}{\sigma/\sqrt{n}}$. Then $T_n(\mu)$ depends on the X's only through the sufficient statistic $\bar{X}$ of μ and its distribution is $N(0, 1)$ for all μ. Construct $(1-\alpha)100\%$ confidence interval of μ and σ^2.

(14 marks)

(c) State

(i) type I and type II error.

(2 marks)

(ii) critical region or rejection region.

(2 marks)

(iii) the power function

(2 marks)

Q4 (a) For a random sample $X_1, X_2, \ldots, X_n$ of Bernoulli (p) variables, it is desired to test

$$H_0: p = 0.49 \text{ versus } H_1: p = 0.51.$$

Use the Central Limit Theorem to determine, approximately, the sample size needed so that the two probabilities of error are both about 0.01. Use a test function that reject H_0 if $\sum_{i=1}^{n} X_i$ is large [$z_{0.01}$=2.33].

(10 marks)

(b) Let $X_1, X_2, \ldots, X_n$ be iid $N(\theta, \sigma^2)$, where θ_0 is a specified value of θ and σ^2 is unknown. We are interested in testing

$$H_0: \theta = \theta_0 \text{ versus } H_1: \theta \neq \theta_0.$$

(i) Show that the test that rejects H_0 when

$$|\bar{X} - \theta_0| > t_{n-1,\alpha/2}\sqrt{\frac{S^2}{n}}$$

is a test of size α.

(5 marks)

(ii) Show that the test in part b(i) can be derived as an Likelihood Ratio Test.

(10 marks)

- END OF QUESTION -