Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2013/2014

COURSE NAME	:	MATHEMATICS FOR ENGINEERING TECHNOLOGY II
COURSE CODE	:	BWM12303
PROGRAMME	:	BNB/BNL
EXAMINATION DATE	:	DECEMBER 2013/JANUARY 2014
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS IN **SECTION A** B) ANSWER **TWO (2)** QUESTIONS IN **SECTION B** C) USE **THREE (3)** DECIMAL PLACES IN YOUR CALCULATIONS

THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

SECTION A

Q1 (a) Find the particular solution for the following initial value problems

$$x^2(1-y)\frac{dy}{dx}+y^2(1+x)=0,\; y(1)=1$$

by using suitable method.

(12 marks)

(b) Use variation parameters method to find the general solutions of the differential equation $y''+y=\cos x$.

(13 marks)

Q2 (a) By using Laplace transform, solve the following initial value problem

$$\frac{d^2y}{dt^2}-2\frac{dy}{dt}+y=e^t,\; y(0)=-2 \text{ and } y'(0)=-3.$$

(12 marks)

(b) Apply fourth-order Runge-Kutta method (RK4) to find the values of $y(0.1)$, $y(0.2)$ and $y(0.3)$ for the following initial value problem

$$\frac{dy}{dx}-y=e^{2x},\quad 0\le x\le 0.3$$

(13 marks)

SECTION B

Q3 (a) Find the following Laplace transform

(i) $L\left\{e^{3t}t^2+4e^{-5t}\right\}$

(ii) $L\left\{\sinh 3t+e^{3t}H(t-3)\right\}$

(12 marks)

(b) Find the invers of the following Laplace Transform

(i) $L^{-1}\left\{\dfrac{16s^2}{(s-3)(s+1)^2}\right\}$

(ii) $L^{-1}\left\{\dfrac{5s}{s^2-4}+\dfrac{6}{(s-2)^2+4}\right\}$

(13 marks)

Q4 (a) Given an ordinary differential equation (ODE) $y''+3y'=6x$.

(i) Show that $y=A+Be^{-3x}+x^2-\frac{2}{3}x$, where A and B are constants, is a general solution of the ordinary differential equation.

(7 marks)

(ii) Hence, find the particular solution of the ODE if $y(0)=2$ and $y'(0)=1/3$.

(7 marks)

(b) Given that $y_1=x^{-1/2}\cos x$ and $y_2=x^{-1/2}\sin x$ are solutions of the equation

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-\frac{1}{4})y=0.$$

Show that the linear combination $y=Ay_1+By_2$, where A and B are constants, is also a solution of the ODE.

(11 marks)

Q5 (a) Solve the following boundary-value problem

$$\frac{d^2y}{dx^2}-\left(1-\frac{x}{5}\right)y=x,\quad y(1)=2 \text{ and } y(3)=-1$$

in the form of matrix equation by using the central finite-difference method with grid size, $h=\Delta x=0.5$.

(15 marks)

(b) Find the particular solution for the following initial value problems

$$(x+y)^2\,dx+(2xy+x^2-1)\,dy=0,\ y(1)=1$$

by using suitable method.

(10 marks)

Q6 Given the initial value problem $\frac{dy}{dx}+3y=e^{2x}$, $y(0)=1$, $0\le x\le 3$. Find the approximate solution for the initial value problem with step size $h=1$ by using:

(a) Euler's method, and

(12 marks)

(b) Second-order Taylor series method.

(13 marks)

- END OF QUESTION -

FINAL EXAMINATION

SEMESTER/SESSION: SEM I/2013/2014
COURSE NAME : MATHEMATICS FOR ENGINEERING TECHNOLOGY II

PROGRAMME:
COURSE CODE: BWM12303

FORMULAE

A. FIRST ORDER ODEs:

1. Separable: $\dfrac{dy}{dx} = f(x,y) \Leftrightarrow v(y)dy = u(x)dx$.
2. Homogeneous: $\dfrac{dy}{dx} = f(x,y) \Leftrightarrow f(\lambda x, \lambda y) = f(x,y)$.

   Hint: Let $y = xv \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$.
3. Linear: $a(x)\dfrac{dy}{dx} + b(x)y = c(x)$. Solution: $y = \dfrac{1}{\rho}\int \rho q(x)dx + A$, where

   $p(x) = \dfrac{b(x)}{a(x)}$, $q(x) = \dfrac{c(x)}{a(x)}$ and $\rho = e^{\int p(x)dx}$.
4. Exact: $\dfrac{dy}{dx} = -\dfrac{M(x,y)}{N(x,y)} \Leftrightarrow M(x,y)dx + N(x,y)dy = 0 \Leftrightarrow \dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$.

   Solution $u = \int Mdx + \phi(y)$.
5. Euler's Method: $\dfrac{dy}{dx} = f(x,y) \Rightarrow y_{i+1} = y_i + h f(x_i, y_i)$.
6. Second Order Taylor Series Method: $\dfrac{dy}{dx} = f(x,y) \Rightarrow y_{i+1} = y_i + h y_i' + \dfrac{h^2}{2!} y_i''$.
7. Fourth-order Runge-Kutta Method (RK4): $y_{i+1} = y_i + \dfrac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

   where $k_1 = h f(x_i, y_i)$, $k_2 = h f(x_i + \dfrac{h}{2}, y_i + \dfrac{k_1}{2})$,

   $k_3 = h f(x_i + \dfrac{h}{2}, y_i + \dfrac{k_2}{2})$, $k_4 = h f(x_i + h, y_i + k_3)$

B. LINEAR SECOND ORDER ODE's: $a y'' + b y' + c y = f(x)$

1. Variation of Parameter: $a y'' + b y' + c y = f(x)$

   Solution: $y = uy_1 + vy_2$, where $u = -\int \dfrac{y_2 f(x)}{aW} dx + A$, $v = \int \dfrac{y_1 f(x)}{aW} dx + B$,

   $W = y_1 y_2' - y_1' y_2$ and y_1 & y_2 are solution of $a y'' + b y' + c y = 0$.

FINAL EXAMINATION

SEMESTER/SESSION: SEM I/2013/2014 PROGRAMME:
COURSE NAME : MATHEMATICS FOR ENGINEERING TECHNOLOGY II COURSE CODE: BWM12303

FORMULAE

2. Finite Difference Method (three-point central difference):

$$y_i' \approx \frac{y_{i+1} - y_i}{2h} \text{ and } y_i'' \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}.$$

C. LAPLACE TRANSFORM

$$L\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
e^{at}	$\frac{1}{s-a}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\sin at$	$\frac{a}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\cos at$	$\frac{s}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$y(t)$	$Y(s)$
t^n, $n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$y'(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$
$t^n f(t)$, $n=1,2,3,\ldots$	$(-1)^n \frac{d^n}{ds^n}F(s)$	$y^n(t)$	$s^nY(s)-s^{n-1}y(0)-s^{n-2}y'(0)-s^{n-3}y''(0)-\ldots-y^{n-1}(0)$