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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2013/2014

COURSE NAME	:	MATHEMATICS FOR ENGINEERING TECHNOLOGY I
COURSE CODE	:	BWM12203
PROGRAMME	:	BNA, BNB, BNC, BND, BNE, BNF, BNG, BNL, BNM, BNN
EXAMINATION DATE	:	DECEMBER 2013/ JANUARY 2014
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL FOUR(4)** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGE

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Q1 (a) Use L' Hopital's rule to find the limits below.

(i) $\lim\limits_{x \to -3} \dfrac{x^2 + 6x + 9}{x + 3}$. (2 maks)

(ii) $\lim\limits_{x \to \infty} \sqrt{x^2 + x} - x$. (5 marks)

(b) (i) Given $y = x^2 y + \cos y$, find $\dfrac{dy}{dx}$. (3 marks)

(ii) Find $\dfrac{dy}{dx}$ of $x = \ln t^2 \sin 3t$ and $y = 5t^3 \cos(3t)$. (4 marks)

(c) Sand is poured onto a growing conical pile of sand, at the constant rate of $6\text{m}^3\text{s}^{-1}$. Suppose the side of the cone makes a 60 degree angle to cone's base. Given $V = \dfrac{1}{3}\pi r^2 h$.

(i) How fast is the height of the sand pile increasing when the volume of the cone is 180m^3? (7 marks)

(ii) If the volume of the cone does not change, find the rate of change of its height, h with respect to its radius, r. (4 marks)

Q2 (a) Evaluate the following integrals.

(i) $\int x^3 \left(x^2 + 1\right)^{-\frac{1}{2}} dx$. (5 marks)

(ii) $\int \sec^5 x \tan x \, dx$. (5 marks)

(b) Evaluate the integral $\int \dfrac{1}{\sqrt{2 - 8x - 4x^2}} dx$ by using the appropriate trigonometric substitution. (9 marks)

(c) Find the area lying between the curves $y = x^2 - 2x$ and $y = 4 - x^2$. (6 marks)

Q3 (a) Determine wether the series $\sum_{n=6}^{\infty}(-1)^{n+1}\frac{n!}{4^n}$ is absolutely convergent, conditionally convergent or divergent. (4 marks)

(b) Given a power series $\sum_{n=1}^{\infty}\frac{(x-3)^n}{n^2}$, determine the interval of convergence and the radius of convergence. (15 marks)

(c) Obtain the Taylor series for $\frac{1}{x}$ in terms of $(x-1)$ up to the fourth power. Then, find the $n-$th terms of the series. (6 marks)

Q4 (a) The position vector of a particle is

$$\mathbf{r}(t)=2t\,\mathbf{i}+t^2\,\mathbf{j} \quad \text{for} \quad -2\le t\le 2.$$

(i) Sketch the graph of $\mathbf{r}(t)$ by indicating the direction of the vector. (5 marks)

(ii) Find the velocity, speed and acceleration of the particle at $t=2$. (7 marks)

(b) Given the vector-valued function

$$\mathbf{r}(t)=\sin t\,\mathbf{i}+\cos t\,\mathbf{j}+2t\,\mathbf{k}.$$

Find its unit tangent vector $\mathbf{T}(t)$, principal unit normal vector $\mathbf{N}(t)$ and curvature and radius of curvature. (13 marks)

- END OF QUESTION -

FINAL EXAMINATION

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Formulae

Indefinite Integrals	Integration of Inverse Functions
$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$	$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C, \ \lvert x\rvert < 1$
$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$	$\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x + C, \ \lvert x\rvert < 1$
$\int \cos x \ dx = \sin x + C$	$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$
$\int \sin x \ dx = -\cos x + C$	$\int \frac{-1}{1+x^2} dx = \cot^{-1} x + C$
$\int \sec^2 x \ dx = \tan x + C$	$\int \frac{1}{\lvert x\rvert \sqrt{x^2-1}} dx = \sec^{-1} x + C, \ \lvert x\rvert > 1$
$\int \csc^2 x \ dx = -\cot x + C$	$\int \frac{-1}{\lvert x\rvert \sqrt{x^2-1}} dx = \csc^{-1} x + C, \ \lvert x\rvert > 1$
$\int \sec x \tan x \ dx = \sec x + C$	$\int \frac{1}{\sqrt{x^2+1}} dx = \sinh^{-1} x + C$
$\int \csc x \cot x \ dx = -\csc x + C$	$\int \frac{1}{\sqrt{x^2-1}} dx = \cosh^{-1} x + C, \ \lvert x\rvert > 1$
$\int e^x dx = e^x + C$	$\int \frac{-1}{\lvert x\rvert \sqrt{1-x^2}} dx = \text{sech}^{-1} \lvert x\rvert + C, \ 0 < x < 1$
$\int \cosh x \ dx = \sinh x + C$	$\int \frac{-1}{\lvert x\rvert \sqrt{1+x^2}} dx = \text{csch}^{-1} \lvert x\rvert + C, \ x \neq 0$
$\int \sinh x \ dx = \cosh x + C$	$\int \frac{1}{1-x^2} dx = \begin{cases} \tanh^{-1} x + C, & \lvert x\rvert < 1 \\ \coth^{-1} x + C, & \lvert x\rvert > 1 \end{cases}$
$\int \text{sech}^2 x \ dx = \tanh x + C$	
$\int \text{csch}^2 x \ dx = -\coth x + C$	
$\int \text{sech}\, x \tanh x \ dx = -\text{sech}\, x + C$	
$\int \text{csch}\, x \coth x \ dx = -\text{csch}\, x + C$	

TRIGONOMETRIC SUBSTITUTION

Expression	*Trigonometry*	*Hyperbolic*
$\sqrt{x^2+k^2}$	$x = k\tan\theta$	$x = k\sinh\theta$
$\sqrt{x^2-k^2}$	$x = k\sec\theta$	$x = k\cosh\theta$
$\sqrt{k^2-x^2}$	$x = k\sin\theta$	$x = k\tanh\theta$

TRIGONOMETRIC SUBSTITUTION

$t = \tan \frac{1}{2}x$		$t = \tan x$	
$\sin x = \dfrac{2t}{1+t^2}$ $\tan x = \dfrac{2t}{1-t^2}$	$\cos x = \dfrac{1-t^2}{1+t^2}$ $dx = \dfrac{2dt}{1+t^2}$	$\sin 2x = \dfrac{2t}{1+t^2}$ $\tan 2x = \dfrac{2t}{1-t^2}$	$\cos 2x = \dfrac{1-t^2}{1+t^2}$ $dx = \dfrac{dt}{1+t^2}$

IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

Trigonometric Functions

$$\cos^2 x + \sin^2 x = 1$$
$$\sin 2x = 2\sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$
$$1 + \tan^2 x = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$
$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$
$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$
$$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$
$$2\sin ax \cos bx = \sin(a+b)x + \sin(a-b)x$$
$$2\sin ax \sin bx = \cos(a-b)x - \cos(a+b)x$$
$$2\cos ax \cos bx = \cos(a-b)x + \cos(a+b)x$$

Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$
$$\cosh x = \frac{e^x + e^{-x}}{2}$$
$$\cosh^2 x - \sinh^2 x = 1$$
$$\sinh 2x = 2\sinh x \cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x$$
$$= 2\cosh^2 x - 1$$
$$= 1 + 2\sinh^2 x$$
$$1 - \tanh^2 x = \operatorname{sech}^2 x$$
$$\coth^2 x - 1 = \operatorname{csch}^2 x$$
$$\tanh 2x = \frac{2\tanh x}{1 + \tanh^2 x}$$
$$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$$
$$\sinh(x \pm y) = \sinh x \cosh y \pm \sinh y \cosh x$$
$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

TAYLOR AND MACLAURIN SERIES

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$
$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

CURVATURE, ARC LENGTH AND SURFACE AREA OF REVOLUTION

Let C is a smooth curve given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, t is parameter, then

the unit tangent vector: $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

the curvature: $\kappa = \dfrac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \dfrac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

the unit normal vector: $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

the radius of curvature: $\rho = 1/\kappa$

the binormal vector: $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$

Arc length

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, $t \in [a,b]$, then the **arc length** $s = \int_a^b \|\mathbf{r}'(t)\|\,dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2}\,dt$

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, $t \in [a,b]$, then the **arc length** $s = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\,dt$