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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2013/2014

COURSE NAME	:	GRAPH THEORY
COURSE CODE	:	BWA 20203
PROGRAMME	:	2BWA
EXAMINATION DATE	:	DECEMBER 2013/JANUARY 2014
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **FIVE (5)** QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

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Q1 (a) State the difference between **trail** and **path**. Hence, define the Eulerian graph and Hamiltonian graph.

(5 marks)

(b) A police car patrols the streets as in Figure **Q1(b)**. The time (in minutes) used by the police car to travel along the street is shown. Knowing that it is the two-way street and the car starts and ends at A. What is the minimum time needed to patrol each street at least once?

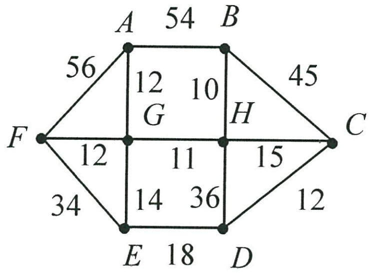


FIGURE Q1(b)

(9 marks)

(c) For the famous Königsberg bridges problem, there are a suggestion to build a new bridge as shown in Figure **Q1(c)** , joining the land areas A and B. Is it possible to cross each of eight bridges exactly once and return to the starting point? Explain your answer.

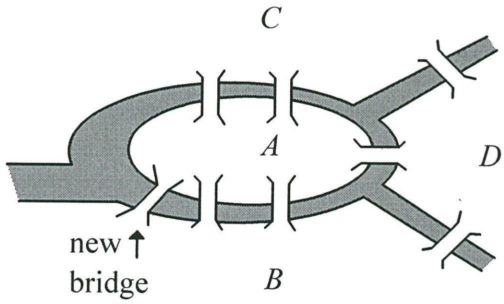


FIGURE Q1(c)

Make a suggestion to show that possible to cross each of eight bridges exactly once and return to the starting point.

(6 marks)

Q2 (a) The Table **Q2(a)** shows the distance in km, between 5 cities. A salesman needs to travel to all the cities from City A and return to City A.

TABLE Q2(a)

	A	B	C	D	E
A	-	14	22	9	12
B	14	-	15	32	14
C	22	15	-	20	13
D	9	32	20	-	25
E	12	14	13	25	-

(i) Use Prim's algorithm and suitable shortcut to find the upper bound for the solution to the travelling salesman problem for this network.

(ii) Find the lower bound for the problem.

(15 marks)

(b) Given the tree, T as in Figure **Q2(b).**

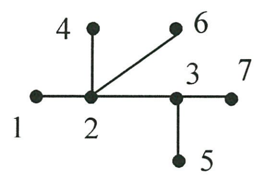


FIGURE Q2(b)

Find the Prüfer sequence.

(5 marks)

Q3 (a) State the **Kuratowski's Theorem.**
Hence, use the Kuratowski's theorem to check if the following graphs in Figure **Q3(a)(i)** and **(ii)** is planar graph or not. Explain your answer. (Remark : draw the planar graph if it is planar graph)

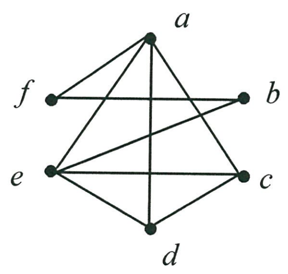


FIGURE Q3(a)(i)

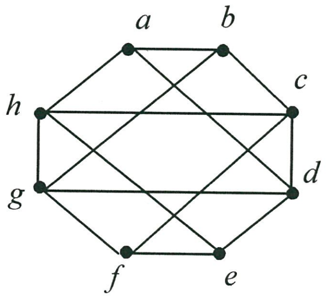


FIGURE Q3(a)(ii)

(8 marks)

(b) Given the planar graph, G as in Figure **Q3(b)**

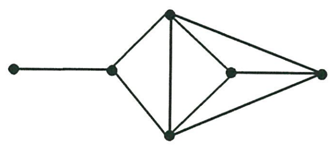


FIGURE Q3(b)

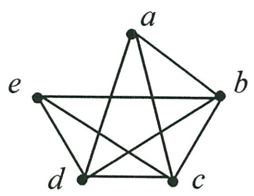
(i) Verify the Euler's formula for the graph G.
(ii) Draw the geometry-dual graph $G*$. Hence, find $n*$ $m*$ and $f*$ for $G*$. What is the relationship between these values with the Euler's formula in (i).

(8 marks)

(c) Check the following statement is TRUE or FALSE. If TRUE, give an example. If FALSE, correct it . [Hint: you may use the graph in (b)]
(i) A graph $G*$ is a geometry dual of graph G is there is a one-one correspondence between the edges of G and those edges of $G*$.
(ii) A graph $G*$ is a geometry dual of graph G if the set of edges in G forms a circuit in G if and only if the corresponding set of edges of $G*$ forms a cutest in $G*$.

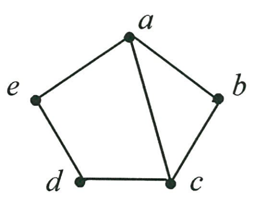
(4 marks)

Q4 (a) For the following graphs.



G_1

FIGURE Q4(a)(i)



G_2

FIGURE Q4(a)(i)

(i) Find the chromatic polynomial for G_1 in Figure **Q4(a)(i).**

(ii) Find the chromatic polynomial for G_2 in Figure **Q4(a)(ii).**

(iii) Find the chromatic numbers for the both graphs above.

(10 marks)

(b) Given the chromatic polynomial for a graph G is

$$P_G(k) = k^6 - 6k^5 + ak^4 - 8k^3 + 7k^2 - 6k$$

(i) Find the value of a.

(ii) How many edges does G have?

(iii) Show that G is a bipartite graph.

(iv) How many ways to colour the graph G in 3 colours?

(6 marks)

(c) Sketch the proof for the following theorem

Theorem : A map G is 2-colourable if and only if G is an Eulerian graph .

(4 marks)

Q5 (a) For the digraph in Figure **Q5(a)**

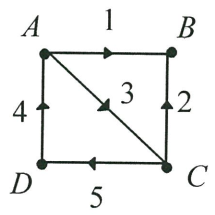


FIGURE Q5(a)

Write the adjacency matrix and incidence matrix.

(3 marks)

(b) Give the definition for **tournament** in digraph.

(1 mark)

(c) For the digraph in Figure **Q5(c)**

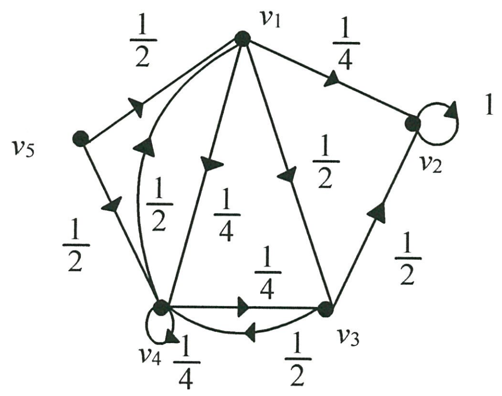


FIGURE Q5(c)

(i) Write the transition matrix.

(ii) What is the condition for the digraph is **strongly-connected**? Is the digraph in Figure **Q5(c)** strongly connected? Explain your answer.

(iii) Is the digraph in Figure **Q5(c) orientable** ? Explain your answer.

(iv) Is the digraph in Figure **Q5(c)** is an **Eulerian digraph**? Explain your answer.

(v) Complete the Table **Q5(c).**

TABLE Q5(c)

State	Persistent/ Transient	Absorbing state(YES/NO)	Periodic/ Aperiodic	Ergodic state (YES/NO)
E_1				
E_2				
E_3				
E_4				
E_5				

Remark : Write down the period if it is periodic.

(16 marks)

Q6 (a) Suppose that three boys x, y, z know three girls a, b, c as in the Table **Q6(a).**

TABLE Q6(a)

Boy	Girls known by boy
x	a, c
y	b, c
z	a

(i) Check the marriage condition for this problem.

(ii) Show that this problem satisfies the following theorem.

Theorem: Let E be a non-empty finite set, and $\zeta = (S_1, \cdots, S_m)$ be a family of non-empty subsets of E. Then ζ has a transversal if and only if the union of any k of the subsets S_i contains at least k elements. $(1 \le k \le m)$

(7 marks)

(b) Verify the König-Egerváry Theorem for the following matrices.

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

By regarding the $(0,1)-$ matrix as the incidence matrix of a family of subsets, determine the respective family has a **transversal** or not. Explain your answer.

(7 marks)

(c) For the graph in Figure **Q6(c)**

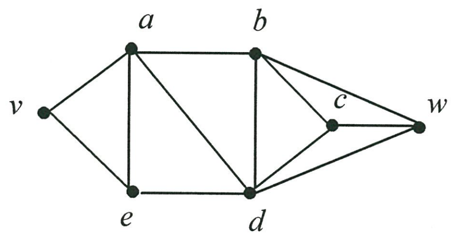


FIGURE Q6(c)

(i) List the vw-disconnecting set and vw-seperating set.

(ii) Use the edge-form of Menger's theorem and vertex-form of Menger's theorem to find the maximum number of edge-disjoint path and vertex-disjoint path respectively. List down the paths.

(6 marks)

- END OF QUESTION -