CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2013/2014

COURSE NAME	:	ENGINEERING MATHEMATICS IV
COURSE CODE	:	BWM30602
PROGRAMME	:	2BEE, 3BEE
EXAMINATION DATE	:	DECEMBER 2013/JANUARY 2014
DURATION	:	2 HOURS 30 MINUTES
INSTRUCTION	:	A) ANSWER **ALL QUESTIONS** B) **ALL** CALCULATIONS AND ANSWERS MUST BE IN THREE DECIMAL PALCES.

THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

Q1 (a) You are working for a start-up computer assembly company and have been asked to determine the minimum number of computers that the shop will have to sell to make a profit. The equation that gives the minimum number of computers n to be sold after considering the total costs and the total sales is

$$f(n) = 40n^{1.5} - 875n + 35000 = 0.$$

Use the Newton-Raphson method for finding roots of the equation (to find the minimum number of computers) that need to be sold to make a profit. Given the initial guess of the root of $f(n) = 0$ as $n_0 = 50$. Conduct three iterations to estimate the root of the above equation.

(12 marks)

(b) Solve the following system of linear equations using Thomas algorithm method.

$$\begin{pmatrix} 2.04 & -1 & 0 & 0 \\ -1 & 2.04 & -1 & 0 \\ 0 & -1 & 2.04 & -1 \\ 0 & 0 & -1 & 2.04 \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{pmatrix} = \begin{pmatrix} 40.8 \\ 0.8 \\ 0.8 \\ 200.8 \end{pmatrix}$$

(13 marks)

Q2 (a) Given matrix A defined by

$$A = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 4 \end{bmatrix}$$

Find the dominant eigenvalue and its corresponding eigenvector for matrix A by using power method. Let the initial guess for eigenvector, $v^{(0)} = (1 \quad 1 \quad 1)^T$. Calculate until $|m_{k+1} - m_k| < 0.005$.

(13 marks)

(b) Use suitable Simpson's rule to evaluate the following integral

$$\int_0^{4.5} \sqrt{x}\, dx$$

by using uniform step size $h = 0.5$.

(12 marks)

Q3 (a) Given the initial value problem (IVP)

$$\frac{dy}{dt} - 2y = -t\,y^2,\; y(0) = 1$$

with step size, $h = 0.2$. Apply fourth-order Runge-Kutta method (RK4) to find the approximate value of $y(0.2)$.

(12 marks)

(b) Solve the boundary-value problem of

$$y'' - \left(1 - \frac{x}{5}\right) y = x,\;\; y(1) = 2 \text{ and } y(3) = -1$$

by using the central finite-difference method with grid size, $h = \Delta x = 0.5$.

(13 marks)

Q4 (a) Let $u(x,t)$ be the displacement of uniform wire which is fixed at both ends along x-axis at time t. The distribution of $u(x,t)$ is given by the wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2},\; 0 < x < 1,\; t > 0$$

with the boundary conditions $u(0,t) = u(1,t) = 0$ and the initial conditions $u(x,0) = \sin 4\pi x$, $\frac{\partial u}{\partial t}(x,0) = 0$ for $0 \le x \le 1$. Solve the wave equation up to level $t = 0.1$ by using finite-difference method with $\Delta x = h = 0.2$ and $\Delta t = k = 0.1$.

(12 marks)

(b) Given the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 1$, and $t > 0$ with the boundary conditions $u(0,t) = 20t^2$, $u(1,t) = 10t$ and the initial conditions $u(x,0) = x(1-x)$ for $0 \le x \le 1$. By using explicit finite-difference method, solve the heat equation for $0 \le t \le 0.03$ with step sizes $\Delta x = h = 0.5$ and $\Delta t = k = 0.01$.

(13 marks)

--END OF QUESTION--

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FORMULAS

Newton-Raphson method: $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}, \quad i = 0,1,2,\dots,n$

Thomas Algorithm:

i	**1**	**2**	**...**	***n***
d_i				
e_i				
c_i				
b_i				
$\alpha_1 = d_1$ $\alpha_i = d_i - c_i\beta_{i-1}$				
$\beta_i = \dfrac{e_i}{\alpha_i}$				
$y_1 = \dfrac{b_1}{\alpha_1}$ $y_i = \dfrac{b_i - c_i y_{i-1}}{\alpha_i}$				
$x_n = y_n$ $x_i = y_i - \beta_i x_{i+1}$				

3-point central difference: $f'(x) \approx \dfrac{f(x+h) - f(x-h)}{2h}$

$$f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

Simpson's $\dfrac{1}{3}$ rule:

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ \text{even}}}^{n-2} f_i \right]$$

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Simpson's $\frac{3}{8}$ rule

$$\int_a^b f(x)dx \approx \frac{3}{8}h\left[(f_0+f_n)+3(f_1+f_2+f_4+f_5+\ldots+f_{n-2}+f_{n-1})+2(f_3+f_6+\ldots+f_{n-3})\right]$$

Power Method $v^{(k+1)} = \frac{1}{m_{k+1}} A v^{(k)}, \quad k = 0,1,2,\ldots$

Classical 4th order Runge-Kutta method. $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$ $\quad k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$ $\quad k_4 = hf(x_i + h, y_i + k_3)$

Boundary value problems: $y_i' \approx \frac{y_{i+1} - y_{i-1}}{2h}$, $y_i'' \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$

Explicit finite difference method (FDM):

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \Leftrightarrow \frac{u_{i,j+1} - u_{i,j}}{k} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \Leftrightarrow \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

$$\frac{\partial u(x,0)}{\partial t} = \frac{u_{i,j+1} - u_{i,j-1}}{2k} = g(x_i)$$