UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2012/2013

COURSE NAME	:	ENGINEERING MATHEMATICS I
COURSE CODE	:	BWM10103 / BSM1913
PROGRAMME	:	1 BEB / 1 BEC / 1 BED / 1 BEF / 1 BEH / 1 BEU / 1 BDD / 2 BFF
EXAMINATION DATE	:	DECEMBER 2012 / JANUARY 2013
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS.

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

Q3 (a) Find $\lim_{x \to 0} \frac{e^x - x - 1}{7x^2}$ by using L'Hôpital's rule.

(4 marks)

(b) Find $\frac{dy}{dx}$ if $\frac{y^4 + x^4}{x^3 y^3} = x^2 y^4$.

(5 marks)

(c) Given parametric function of $x = 1 - 3\sin(4\pi t)$ and $y = 2 + 3\cos(4\pi t)$.

Show that $\frac{d^2 y}{dx^2} = -\frac{1}{3}\sec^3(4\pi t)$.

(5 marks)

(d) Sand falls to the ground at a constant rate of 110 ft^3/ min forming a cone shape with radius that is always three times its height.

(i) Find the volume relationship of the sand fall, V in terms of h.

[Hint: Volume, $V = \frac{1}{3}\pi r^2 h$]

(ii) Hence, determine the height of sand's changing rate, $\frac{dh}{dt}$ when its height, $h = 10$ ft.

(6 marks)

Q4 (a) Evaluate $\int (1-x)\sqrt{x}\,dx$.

(3 marks)

(b) Evaluate $\int \sin^3 x\,dx$ by using appropriate trigonometric identity.

(7 marks)

(c) By using the substitution $t = \tan\frac{x}{2}$, evaluate $\int \sec x\,dx$.

(10 marks)

Q5 (a) Solve the following

(i) $\frac{d}{dx}\left[\sin^{-1}(\ln x)\right]$.

(ii) $\int \frac{3}{\sqrt{x^2-9}}\,dx$. [Hint: Apply hyperbolic substitution.]

(9 marks)

(b) Find the curvature of the ellipse $x = 3\cos t$ and $y = 2\sin t$ at the points corresponding to $t = 0$ and $t = \pi/2$.

(4 marks)

(c) The curve $y = \sqrt{4-x^2}$, $-1 \le x \le 1$, is an arc of the circle $x^2 + y^2 = 4$. Find the area of the surface obtained by rotating this arc 360° about the x-axis.

(6 marks)

- END OF QUESTION -

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Formulae

Indefinite Integrals

$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$

$\int \frac{1}{x}\, dx = \ln|x| + C$

$\int \cos x\, dx = \sin x + C$

$\int \sin x\, dx = -\cos x + C$

$\int \sec^2 x\, dx = \tan x + C$

$\int \csc^2 x\, dx = -\cot x + C$

$\int \sec x \tan x\, dx = \sec x + C$

$\int \csc x \cot x\, dx = -\csc x + C$

$\int e^x dx = e^x + C$

$\int \cosh x\, dx = \sinh x + C$

$\int \sinh x\, dx = \cosh x + C$

$\int \text{sech}^2 x\, dx = \tanh x + C$

$\int \text{csch}^2 x\, dx = -\coth x + C$

$\int \text{sech}\, x \tanh x\, dx = -\text{sech}\, x + C$

$\int \text{csch}\, x \coth x\, dx = -\text{csch}\, x + C$

Integration of Inverse Functions

$\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x + C, \; |x| < 1$

$\int \frac{-1}{\sqrt{1-x^2}}\, dx = \cos^{-1} x + C, \; |x| < 1$

$\int \frac{1}{1+x^2}\, dx = \tan^{-1} x + C$

$\int \frac{-1}{1+x^2}\, dx = \cot^{-1} x + C$

$\int \frac{1}{|x|\sqrt{x^2-1}}\, dx = \sec^{-1} x + C, \; |x| > 1$

$\int \frac{-1}{|x|\sqrt{x^2-1}}\, dx = \csc^{-1} x + C, \; |x| > 1$

$\int \frac{1}{\sqrt{x^2+1}}\, dx = \sinh^{-1} x + C$

$\int \frac{1}{\sqrt{x^2-1}}\, dx = \cosh^{-1} x + C, \; |x| > 1$

$\int \frac{-1}{|x|\sqrt{1-x^2}}\, dx = \text{sech}^{-1} |x| + C, \; 0 < x < 1$

$\int \frac{-1}{|x|\sqrt{1+x^2}}\, dx = \text{csch}^{-1} |x| + C, \; x \neq 0$

$\int \frac{1}{1-x^2}\, dx = \begin{cases} \tanh^{-1} x + C, & |x| < 1 \\ \coth^{-1} x + C, & |x| > 1 \end{cases}$

TAYLOR AND MACLAURIN SERIES

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

TRIGONOMETRIC SUBSTITUTION

Expression	***Trigonometry***	***Hyperbolic***
$\sqrt{x^2+k^2}$	$x = k\tan\theta$	$x = k\sinh\theta$
$\sqrt{x^2-k^2}$	$x = k\sec\theta$	$x = k\cosh\theta$
$\sqrt{k^2-x^2}$	$x = k\sin\theta$	$x = k\tanh\theta$

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Formulae

TRIGONOMETRIC SUBSTITUTION

$t=\tan\frac{1}{2}x$		$t=\tan x$	
$\sin x=\dfrac{2t}{1+t^2}$	$\cos x=\dfrac{1-t^2}{1+t^2}$	$\sin 2x=\dfrac{2t}{1+t^2}$	$\cos 2x=\dfrac{1-t^2}{1+t^2}$
$\tan x=\dfrac{2t}{1-t^2}$	$dx=\dfrac{2dt}{1+t^2}$	$\tan 2x=\dfrac{2t}{1-t^2}$	$dx=\dfrac{dt}{1+t^2}$

IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

Trigonometric Functions

$$\cos^2 x+\sin^2 x=1$$
$$\sin 2x=2\sin x\cos x$$
$$\cos 2x=\cos^2 x-\sin^2 x$$
$$=2\cos^2 x-1$$
$$=1-2\sin^2 x$$
$$1+\tan^2 x=\sec^2 x$$
$$1+\cot^2 x=\csc^2 x$$
$$\tan 2x=\frac{2\tan x}{1-\tan^2 x}$$
$$\tan(x\pm y)=\frac{\tan x\pm\tan y}{1\mp\tan x\tan y}$$
$$\sin(x\pm y)=\sin x\cos y\pm\sin y\cos x$$
$$\cos(x\pm y)=\cos x\cos y\mp\sin x\sin y$$
$$2\sin ax\cos bx=\sin(a+b)x+\sin(a-b)x$$
$$2\sin ax\sin bx=\cos(a-b)x-\cos(a+b)x$$
$$2\cos ax\cos bx=\cos(a-b)x+\cos(a+b)x$$

Hyperbolic Functions

$$\sinh x=\frac{e^x-e^{-x}}{2}$$
$$\cosh x=\frac{e^x+e^{-x}}{2}$$
$$\cosh^2 x-\sinh^2 x=1$$
$$\sinh 2x=2\sinh x\cosh x$$
$$\cosh 2x=\cosh^2 x+\sinh^2 x$$
$$=2\cosh^2 x-1$$
$$=1+2\sinh^2 x$$
$$1-\tanh^2 x=\operatorname{sech}^2 x$$
$$\coth^2 x-1=\operatorname{csch}^2 x$$
$$\tanh 2x=\frac{2\tanh x}{1+\tanh^2 x}$$
$$\tanh(x\pm y)=\frac{\tanh x\pm\tanh y}{1\pm\tanh x\tanh y}$$
$$\sinh(x\pm y)=\sinh x\cosh y\pm\sinh y\cosh x$$
$$\cosh(x\pm y)=\cosh x\cosh y\pm\sinh x\sinh y$$

CURVATURE, ARC LENGTH AND SURFACE AREA OF REVOLUTION

$$\kappa=\frac{\left|\dfrac{d^2y}{dx^2}\right|}{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}$$

$$\kappa=\frac{|\dot{x}\ddot{y}-\dot{y}\ddot{x}|}{\left[\dot{x}^2+\dot{y}^2\right]^{3/2}}$$

$$L=\int_{x_1}^{x_2}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$L=\int_{t_1}^{t_2}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

$$L=\int_{y_1}^{y_2}\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$S=2\pi\int_{x_1}^{x_2}f(x)\sqrt{1+\left(\frac{d}{dx}[f(x)]\right)^2}\,dx$$

$$S=2\pi\int_{y_1}^{y_2}g(y)\sqrt{1+\left(\frac{d}{dy}[g(y)]\right)^2}\,dy$$

Q1 (a) Using the appropriate test determine whether each of the following series converges or diverges.

(i) $\sum_{n=1}^{\infty} \frac{n^2}{n^2+3}$.

(ii) $\sum_{n=1}^{\infty} \frac{100^n}{n!}$.

(iii) $\sum_{n=1}^{\infty} \left(\frac{n+1}{n}\right)^{n^2}$.

(8 marks)

(b) Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^n}{n}$.

(6 marks)

(c) Show that the Maclaurin series for $\cos x$ up to x^6 is

$$1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}.$$

Then approximate $\int_0^1 \cos(x^2)\, dx$ to three decimal places.

(6 marks)

Q2 (a) Find

(i) $\lim_{x \to c} \frac{x^2 - 2cx + c^2}{x - c}$

(ii) $\lim_{x \to +\infty} \frac{2e^x + 5e^{3x}}{e^{2x} - e^{3x}}$

(iii) $\lim_{x \to +\infty} 3x - \sqrt{x^4 - 2x + 3}$

(10 marks)

(b) Determine the point(s) at which the following functions have discontinuities.

(i) $f(x) = 4x^5 - 3x^2 + 1$

(ii) $f(x) = \frac{x+1}{x^2 - 4}$

(4 marks)

(c) Let

$$f(x) = \begin{cases} \left(\frac{x+5}{x}\right)^3 & , \quad 0 < x < 5 \\ 8 & , \quad x = 5 \\ \sqrt{x^2 + 24} + 1 & , \quad 5 < x < 10 \end{cases}$$

Determine whether $f(x)$ is continuous or not at $x = 5$.

(6 marks)