UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER 2
SESSION 2012/2013

COURSE NAME	:	MATHEMATICS FOR ENGINEERING TECHNOLOGY II
COURSE CODE	:	BWM 12303
PROGRAMME	:	2 BNB / 2BND / 2BNK / 2BNL / 2BNN
EXAMINATION DATE	:	JUNE 2013
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS EXAMINATION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 Obtain a numerical solution of the equation

$$\frac{dy}{dx}=\frac{e^{2x}-3y}{2}, \quad y(0)=1$$

at constant intervals of $x = 0.2$ for

(a) the range from $x=0$ to $x=1$, by the second-order Taylor series method.

(12 marks)

(b) $y(0.2)$ and $y(0.4)$, by the fourth-order Runge-Kutta Method.

(13 marks)

Q2 Given $f(t)=\begin{cases}\sin t, & 0\le t<\pi \\ 0, & t\ge\pi.\end{cases}$

(a) Show that $\mathcal{L}\{f(t)\}=\frac{1}{s^2+1}+\frac{e^{-\pi s}}{s^2+1}$.

(5 marks)

(b) Show, by using convolution theorem, that $\mathcal{L}^{-1}\left\{\frac{1}{(s^2+1)^2}\right\}=\frac{1}{2}(\sin t-t\cos t)$.

[Hint: $2\sin A\sin B=\cos(A-B)-\cos(A+B)$]

(7 marks)

(c) Solve the initial value problem

$$y''+y=f(t),$$

where $y(0)=1$ and $y'(0)=-1$.

(13 marks)

Q3 (a) A particle drops vertically downward with a velocity v_0. If the velocity of the particle at time t is v and its acceleration is given by

$$\frac{dv}{dt}=g-kv,$$

where g is acceleration due to gravity and k is a constant, show that

$$v=\frac{g}{k}-\left(\frac{g}{k}-v_0\right)e^{-kt}.$$

Find the limiting velocity $(t\to\infty)$ of the particle.

(12 marks)

(b) Given $\left(y\cos x + 2xe^{y}\right)dx + \left(\sin x + x^2e^{y} + 2\right)dy = 0.$

Show that the equation is exact and then solve the differential equation.

(13 marks)

Q4 (a) Find by using the method of undetermined coefficients, the solution of the differential equation

$$\frac{d^2y}{dx^2} + 4y = e^{-2x} + \cos 2x$$

which satisfies the conditions $y = 1$ and $\frac{dy}{dx} = 2,$ when $x = 0.$

(13 marks)

(b) Use the method of variation of parameters to find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{e^x}{1+e^x}.$$

(12 marks)

- END OF QUESTION -

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FORMULAE

Second-order Differential Equation

The roots of characteristic equation and the general solution for differential equation $ay'' + by' + cy = 0$.

Characteristic equation: $am^2 + bm + c = 0$.		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y = Ae^{m_1 x} + Be^{m_2 x}$
2.	Real and equal roots: $m = m_1 = m_2$	$y = (A + Bx)e^{mx}$
3.	Complex roots: $m_1 = \alpha + \beta i$, $m_2 = \alpha - \beta i$	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

The method of undetermined coefficients

For non-homogeneous second order differential equation $ay'' + by' + cy = f(x)$, the particular solution is given by $y_p(x)$:

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + A_{n-1}x^{n-1} + \cdots + A_1 x + A_0$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(P\cos\beta x + Q\sin\beta x)$
$P_n(x)e^{\alpha x}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)e^{\alpha x}$
$P_n(x)\begin{cases}\cos\beta x \\ \sin\beta x\end{cases}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)\cos\beta x +$ $x^r(C_n x^n + C_{n-1}x^{n-1} + \cdots + C_1 x + C_0)\sin\beta x$
$Ce^{\alpha x}\begin{cases}\cos\beta x \\ \sin\beta x\end{cases}$	$x^r e^{\alpha x}(P\cos\beta x + Q\sin\beta x)$
$P_n(x)e^{\alpha x}\begin{cases}\cos\beta x \\ \sin\beta x\end{cases}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)e^{\alpha x}\cos\beta x +$ $x^r(C_n x^n + C_{n-1}x^{n-1} + \cdots + C_1 x + C_0)e^{\alpha x}\sin\beta x$

Note : r is the least non-negative integer (r = 0, 1, or 2) which determine such that there is no terms in particular integral $y_p(x)$ corresponds to the complementary function $y_c(x)$.

The method of variation of parameters

The general solution for $ay'' + by' + cy = f(x)$ is

$$y = y_h + y_p,$$

where $y_h = Ay_1 + By_2$ (homogeneous solution), $y_p = uy_1 + vy_2$ (particular solution), and

$$u = -\int \frac{y_2 f(x)}{aW}dx, \qquad v = \int \frac{y_1 f(x)}{aW}dx \text{ and } W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'.$$

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Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
A	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
e^{at}	$\frac{1}{s-a}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\sin at$	$\frac{a}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\cos at$	$\frac{s}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$y(t)$	$Y(s)$
$t^n,\ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$y'(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$
$t^n f(t),\ n=1,2,3,\ldots$	$(-1)^n \frac{d^n}{ds^n}F(s)$		

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Numerical solution for ordinary differential equations

Initial value problems:

Euler's method: $y(x_{i+1}) = y(x_i) + hy'(x_i)$

Second order Taylor series method:

$$y(x_{i+1}) = y(x_i) + hy'(x_i) + \frac{h^2}{2!}y''(x_i)$$

Improved Euler's method (Mid-point method):

$$y_{i+1} = y_i + k_2$$

where $k_1 = hf(x_i, y_i)$

$$k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$$

Heun's method:

$$y_{i+1} = y_i + \frac{1}{4}k_1 + \frac{3}{4}k_2$$

where $k_1 = hf(x_i, y_i)$

$$k_2 = hf(x_i + \frac{2}{3}h, y_i + \frac{2}{3}k_1)$$

Modified Euler's method:

$$y_{i+1} = y_i + \frac{1}{2}k_1 + \frac{1}{2}k_2$$

where $k_1 = hf(x_i, y_i)$

$$k_2 = hf(x_i + h, y_i + k_1)$$

Classic 4th order Runge-Kutta method:

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where $k_1 = hf(x_i, y_i)$

$$k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$$

$$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$$

$$k_4 = hf(x_i + h, y_i + k_3)$$

Boundary value problems:

Finite difference method:

$$y_i' \approx \frac{y_{i+1} - y_{i-1}}{2h}$$

$$y_i'' \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$