UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2012/2013

COURSE NAME	:	ENGINEERING MATHEMATICS IV
COURSE CODE	:	BWM 30603 / BSM 3913
PROGRAMME	:	2 BDD / BFF 3 BDD / BFF 4 BDD / BEE / BFF
EXAMINATION DATE	:	JUNE 2013
DURATION	:	3 HOURS
INSTRUCTIONS	:	ANSWER **ALL** QUESTIONS IN **PART A** AND **TWO (2)** QUESTIONS IN **PART B.** ALL CALCULATIONS AND ANSWERS MUST BE IN THREE (3) DECIMAL PLACES.

THIS QUESTION PAPER CONSISTS OF ***EIGHT (8)*** *PAGES*

PART A

Q1 A thin metal rod with a length of 1 meter and a thermal diffusivity of $c^2 = 2$ is held at a temperature of **zero** at both end points and is insulated from its surroundings everywhere else (see **Figure Q1**). Assume that the left-most point is at the origin. At time $t = 0$, it has a temperature profile of $30\sin(\pi x) + 10\sin(3\pi x)$.

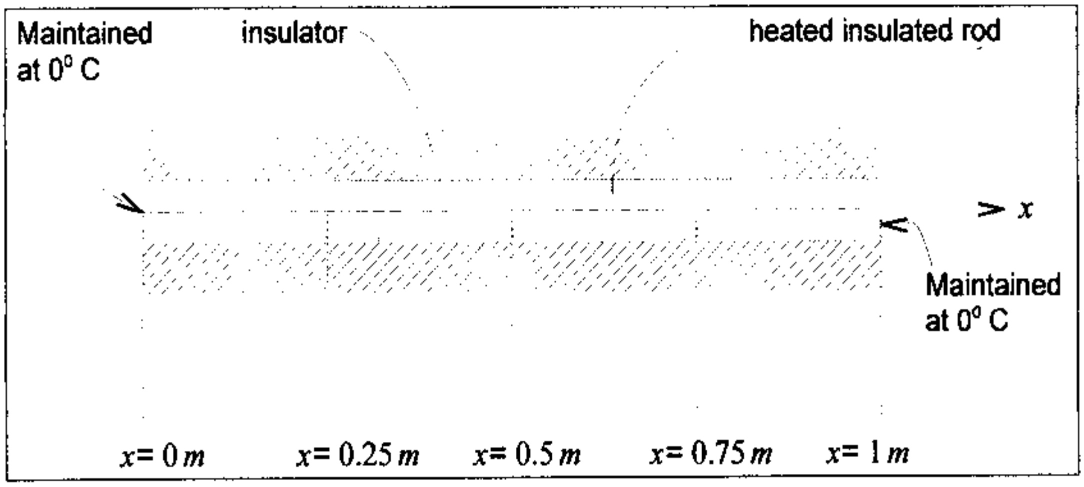

Figure Q1

Find the temperature profile for the rod with $\Delta x = h = 0.25$ and $\Delta t = k = 0.01$ for $0 \le t \le 0.02$ by

(a) writing down the partial differential equation that governs the above problem. (2 marks)

(b) writing down the initial condition. (2 marks)

(c) writing down the boundary conditions. (3 marks)

(d) using explicit method. (9 marks)

(e) using implicit method. (9 marks)

Q2 An aluminum strap with a thickness of 6 mm and the profile shown in **Figure Q2** is to carry a load of 2500 N. Given that the modulus elasticity of material (E) is 70 GPa.

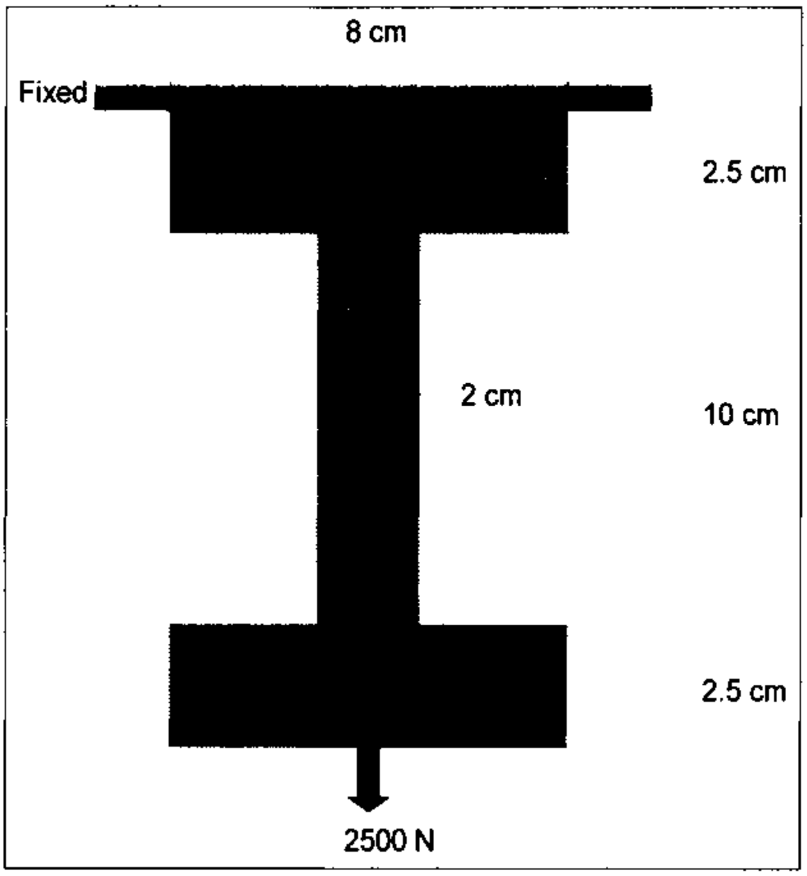

Figure Q2: Loaded Aluminium Strap

(a) Divide the strap into three (3) axial elements, and draw the elements indicating the nodes and elements numbers. Your analysis in the next question should be consistently based on your node and element definitions.

(5 marks)

(b) Do a finite element analysis to determine the deflection at sectional points A, B and C. You have to use direct elimination method on handling the constraints. The deflection unit must be in micro meter (10^{-6} m).

(20 marks)

PART B

Q3 (a) You are working for a start-up computer assembly company and have been asked to determine the minimum number of computers that the shop will have to sell to make a profit. The equation that gives the minimum number of computers n to be sold after considering the total costs and the total sales is

$$f(n) = 40n^{1.5} - 875n + 35000 = 0.$$

Use the Newton-Raphson method for finding roots of equations to find the minimum number of computers that need to be sold to make a profit. Given the initial guess of the root of $f(n) = 0$ as $n_0 = 50$. Conduct three iterations to estimate the root of the above equation.

(10 marks)

(b) Solve the following tridiagonal system using Thomas algorithm method.

$$\begin{pmatrix} 2.04 & -1 & 0 & 0 \\ -1 & 2.04 & -1 & 0 \\ 0 & -1 & 2.04 & -1 \\ 0 & 0 & -1 & 2.04 \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{pmatrix} = \begin{pmatrix} 40.8 \\ 0.8 \\ 0.8 \\ 200.8 \end{pmatrix}$$

(15 marks)

Q4 (a) Compute $f(0.3)$ for the data in **Table Q4(a)** by using Newton's divided difference formula.

Table Q4(a)

x	0	1	3	4	7
$f(x)$	1	3	49	129	813

(8 marks)

(b) **Table Q4(b)** gives the velocity, v of an object at various points in time, t.

Table Q4(b)

Time, t (sec.)	3	5	7	9	11
Velocity, v (m/sec.)	4.6	8.030	11.966	16.885	19.904

By taking $h = 2$ seconds, estimate the acceleration at the time $t = 5$ seconds by using **ALL APPROPRIATE** difference formulas.

(8 marks)

(c) Use suitable Simpson's rule to approximate $\int_0^{4.5} \sqrt{x}\, dx$ using a regular partition with $n = 9$.

(9 marks)

Q5 (a) Given

$$A = \begin{pmatrix} 1.5 & 3 & 2 \\ 0 & 2.3 & 1 \\ 0 & 3.4 & 1.5 \end{pmatrix}.$$

By taking $v^{(0)} = (1 \quad 1 \quad 0)^T$, calculate the smallest eigenvalue and its corresponding eigenvector by using inverse power method. Iterate until $|m_{k+1} - m_k| < 0.005$.

(7 marks)

(b) Given the initial-value problem (IVP) as follows:

$$y' = 4e^{0.8x} - 0.5y, \quad y(0) = 2.$$

Approximate the solution at $x = 1$ with step size of $h = 1$ by using fourth-order Runge-Kutta method.

(10 marks)

(c) Solve the boundary-value problem (BVP), $y'' - xy' + 3y = 11x$ with conditions $y(0) = 1$ and $y(1) = 2$ where $h = 0.25$ by using finite-difference method.

(8 marks)

- END OF QUESTION -

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FORMULAS

Nonlinear equations

Newton-Raphson method: $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}, \quad i = 0,1,2,...,n$

System of linear equations

Thomas Algorithm:

i	1	2	...	n
d_i				
e_i				
c_i				
b_i				
$\alpha_1 = d_1$ $\alpha_i = d_i - c_i\beta_{i-1}$				
$\beta_i = \dfrac{e_i}{\alpha_i}$				
$y_1 = \dfrac{b_1}{\alpha_1}$ $y_i = \dfrac{b_i - c_i y_{i-1}}{\alpha_i}$				
$x_n = y_n$ $x_i = y_i - \beta_i x_{i+1}$				

Interpolation

Newton's divided difference:

$$P_n(x) = f_0^{[0]} + f_0^{[1]}(x - x_0) + f_0^{[2]}(x - x_0)(x - x_1) + \ldots + f_0^{[n]}(x - x_0)(x - x_1)\ldots(x - x_{n-1})$$

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Numerical differentiation and integration

Differentiation:
First derivatives:

2-point forward difference: $f'(x) \approx \dfrac{f(x+h)-f(x)}{h}$

2-point backward difference: $f'(x) \approx \dfrac{f(x)-f(x-h)}{h}$

3-point forward difference: $f'(x) \approx \dfrac{-f(x+2h)+4f(x+h)-3f(x)}{2h}$

3-point backward difference: $f'(x) \approx \dfrac{3f(x)-4f(x-h)+f(x-2h)}{2h}$

3-point central difference: $f'(x) \approx \dfrac{f(x+h)-f(x-h)}{2h}$

5-point difference: $f'(x) \approx \dfrac{-f(x+2h)+8f(x+h)-8f(x-h)+f(x-2h)}{12h}$

Integration:

Simpson's $\frac{1}{3}$ rule: $\displaystyle\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1\\ i\,\text{odd}}}^{n-1} f_i + 2\sum_{\substack{i=2\\ i\,\text{even}}}^{n-2} f_i\right]$

Simpson's $\frac{3}{8}$ rule:

$$\int_a^b f(x)\,dx \approx \frac{3}{8}h\left[f_0 + f_n + 3(f_1 + f_2 + f_4 + f_5 + \dots + f_{n-2} + f_{n-1}) + 2(f_3 + f_6 + \dots + f_{n-3}\right]$$

Eigenvalue

Power Method: $v^{(k+1)} = \dfrac{1}{m_{k+1}} A v^{(k)}, \quad k = 0,1,2,\dots$

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Ordinary differential equations

Initial value problems:

Mid-point method: $y_{i+1} = y_i + k_2$

where $k_1 = hf(x_i, y_i)$, $k_2 = hf\left(x_i + \frac{h}{2},\ y_i + \frac{k_1}{2}\right)$

Fourth-order Runge-Kutta method: $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$, $k_2 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right)$

$k_3 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right)$, $k_4 = hf(x_i + h, y_i + k_3)$

Boundary value problems:

Finite-difference method:

$$y_i' \approx \frac{y_{i+1} - y_{i-1}}{2h}, \qquad y_i'' \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

Partial differential equation

Heat Equation: Explicit finite difference method

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \quad \Rightarrow \quad \frac{u_{i,j+1} - u_{i,j}}{k} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

Heat equation: Implicit Crank-Nicolson

$$\left(\frac{\partial u}{\partial t}\right)_{i,j+\frac{1}{2}} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j+\frac{1}{2}} \quad \Rightarrow \quad \frac{u_{i,j+1} - u_{i,j}}{k} = \frac{c^2}{2}\left(\frac{u_{i+1,j+1} - 2u_{i,j+1} + u_{i-1,j+1}}{h^2} + \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2}\right)$$