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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2012/2013

COURSE NAME	:	ENGINEERING MATHEMATICS IV
COURSE CODE	:	BWM 30602
PROGRAMME	:	2 BEE, 3 BEE
EXAMINATION DATE	:	JUNE 2013
DURATION	:	2 HOURS AND 30 MINUTES
INSTRUCTION	:	1. ANSWER **ALL** QUESTIONS 2. ALL ANSWERS MUST BE IN **THREE (3) DECIMAL PLACES**.

THIS QUESTION PAPER CONSISTS OF **FOUR (4)** PAGES

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Q3 (a) Apply fourth-order Runge-Kutta method (RK4) to find the values of $y(0.1)$, $y(0.2)$ and $y(0.3)$ for the following initial value problem

$$\frac{dy}{dx} - y = e^{2x}, \quad 0 \le x \le 0.3$$

with initial value $y(0) = -1$ and step size, $h = 0.1$.

(10 marks)

(b) Solve the boundary-value problem of

$$\frac{d^2y}{dx^2} - \left(1 - \frac{x}{5}\right)y = x, \quad y(1) = 2 \text{ and } y(3) = -1$$

by using the central finite-difference method with grid size, $h = \Delta x = 0.5$.

(15 marks)

Q4 (a) Let $u(x,t)$ be the displacement of uniform wire which is fixed at both ends along x-axis at time t. The distribution of $u(x,t)$ is given by the wave equation

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \; 0 < t < 0.5$$

with the boundary conditions $u(0,t) = u(1,t) = 0$ and the initial conditions $u(x,0) = \sin \pi x$, $\frac{\partial u}{\partial t}(x,0) = 0$ for $0 \le x \le 1$. Solve the wave equation up to level $t = 0.1$ by using finite-difference method with $\Delta x = h = 0.25$ and $\Delta t = k = 0.1$.

(10 marks)

(b) Use finite-difference method to solve the Laplace's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \; 1 < x < 2, \text{ and } 0 < y < 1$$

with the boundary conditions are $u(x,0) = 2\ln x$, $u(x,1) = \ln(x^2+1)$, for $1 \le x \le 2$ and $u(1,y) = \ln(y^2+1)$, $u(2,y) = \ln(y^2+4)$ for $0 \le y \le 1$, with step size $\Delta x = \Delta y = 1/3$. Find the error, if the exact solution is $u(x,y) = \ln(x^2+y^2)$.

(15 marks)

--END OF QUESTION--

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FORMULAS

Newton-Raphson method $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}$. $i = 0,1,2,3,...$

Simpson's $\frac{3}{8}$ rule:

$$\int_a^b f(x)dx \approx \frac{3}{8}h\left[f_0 + f_n + 3(f_1 + f_2 + f_4 + f_5 + ... + f_{n-2} + f_{n-1}) + 2(f_3 + f_6 + ... + f_{n-3})\right]$$

Power Method $v^{(k+1)} = \dfrac{1}{m_{k+1}} A v^{(k)}, \quad k = 0,1,2,...$

Fourth Order Runge-Kutta method. $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$ $\qquad k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$ $\qquad k_4 = hf(x_i + h, y_i + k_3)$

Boundary value problems: $y_i' \approx \dfrac{y_{i+1} - y_{i-1}}{2h}, \quad y_i'' \approx \dfrac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$

Central finite difference method:

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \quad \Leftrightarrow \quad \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

$$\frac{\partial u(x,0)}{\partial t} = \frac{u_{i,j+1} - u_{i,j-1}}{2k} = g(x_i)$$

Central finite difference for Laplace Equations

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \Leftrightarrow \quad \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2} + \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} = 0$$

Q1 (a) Use the Newton-Raphson method to estimate the root of $e^{-x} - x = 0$ by employing an initial guess of $x_0 = 0$. Iterate until $|f(x_i)| < \varepsilon = 0.005$.

(12 marks)

(b) Solve the following system of linear equations by Gauss elimination method:

$$\begin{aligned} 2.04x_1 - x_2 &= 40.8 \\ -x_1 + 2.04x_2 - x_3 &= 0.8 \\ -x_2 + 2.04x_3 - x_4 &= 0.8 \\ -x_3 + 2.04x_4 &= 200.8 \end{aligned}$$

(13 marks)

Q2 (a) Given matrix A defined by

$$A = \begin{bmatrix} 3 & -1 & 0 \\ -2 & 4 & -3 \\ 0 & -1 & 1 \end{bmatrix}$$

Find the dominant eigenvalue and its corresponding eigenvector for matrix A by using power method. Use initial guess for eigenvector, $v^{(0)} = (1 \quad 1 \quad 1)^T$. Calculate until $|m_{k+1} - m_k| < 0.005$.

(15 marks)

(b) Use Simpson's $\frac{3}{8}$ rule to integrate

$$f(x) = 1 - e^{-2x}$$

from $a = 0$ to $b = 2.5$ with 9 subintervals ($n = 9$).

(10 marks)