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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2012/2013

COURSE NAME	:	ENGINEERING MATHEMATICS II E
COURSE CODE	:	BWM 10303 / BSM1933
PROGRAMME	:	1BEB/C/D/E/H/L/U, 2BEC/D/H/U, 3BEB/C/H, 4BEE
EXAMINATION DATE	:	JUNE 2013
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **FIVE (5)** QUESTIONS FROM SIX (6) QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

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Q1 Given $y'' - xy = 0$.

(a) By assuming $y = \sum_{m=0}^{\infty} c_m x^m$, show that the differential equation $y'' - xy = 0$ can be expressed as

$$\sum_{m=2}^{\infty} m(m-1)\, c_m x^{m-2} - x\left(\sum_{m=0}^{\infty} c_m x^m\right) = 0.$$

(4 marks)

(b) Hence, by shifting the indices, show that the reccurance relation is given by

$$c_{n+2} = \frac{c_{n-1}}{(n+2)(n+1)}, \quad n = 1, 2, 3, \ldots$$

(8 marks)

(c) Then, deduce the coefficient of series for c_n, $n = 0, 1, 2, 3, 4 \ldots 7$ in terms of c_0 and c_1.

(6 marks)

(d) Hence, show that the general solution of the differential equation is

$$y = c_0\left[1 + \frac{1}{3\cdot 2}x^3 + \frac{1}{6\cdot 5\cdot 3\cdot 2}x^6 + \cdots\right] + c_1\left[x + \frac{1}{4\cdot 3}x^4 + \frac{1}{7\cdot 6\cdot 4\cdot 3}x^7 + \cdots\right].$$

(2 marks)

Q2 (a) Given the following periodic function,

$$f(x) = 2x, \qquad -\pi < x < \pi,$$
$$f(x) = f(x + 2\pi).$$

(i) Sketch the periodic function above for the interval $[-3\pi, 3\pi]$.

(ii) Determine whether the above periodic function is an odd function, even function or neither odd nor even function.

(iii) Determine the Fourier series expansion to represent the above periodic function.

(11 marks)

(b) By definition of Fourier transforms, evaluate $\mathcal{F}\{-3\delta(t+2)\}$.

(3 marks)

(c) By referring to the Fourier transform pair table, evaluate

(i) $\mathcal{F}\left\{e^{-3t} H(t)\right\}$.

(ii) $\mathcal{F}\left\{e^{-4t} \sin(\pi t) H(t)\right\}$.

(6 marks)

Q3 (a) Obtain the inverse Laplace transform for $F(s) = \dfrac{4s}{(s-1)(s+1)^2}$.

(7 marks)

(b)

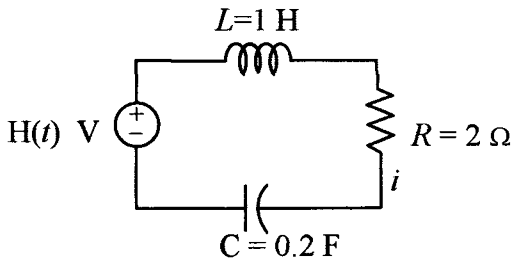

Figure Q3

Show that the governing equation for the circuit in Figure Q3 is given by

$$\frac{di}{dt} + 2i + 5\int_0^t i\,dt = H(t)$$

where $H(t)$ is the unit step function.
Hence, find the current $i(t)$ if there is no initial stored energy.

(13 marks)

Q4 (a) Evaluate $\mathcal{L}\left\{(t + t^2 + \frac{1}{6}t^3)e^{-t}\right\}$.

(5 marks)

(b) Find the Laplace transform of

$$f(t) = \begin{cases} t, & t < 2 \\ t^2, & t \geq 2 \end{cases}$$

by changing it to unit step function first.

(9 marks)

(c) By using the transform of integral, evaluate

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+4)}\right\}$$

(6 marks)

Q5

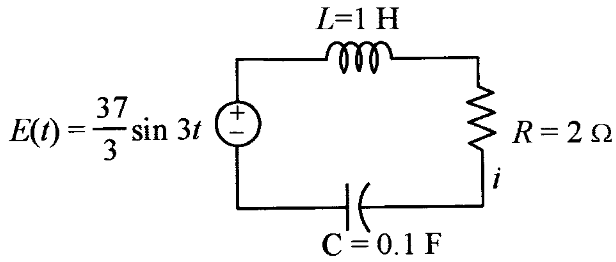

Figure Q5

The *RLC* circuit in the Figure Q5 consists of a resistor, *R*, an inductor, *L* and a capacitor, *C* connected in series together with a voltage source, $\frac{37}{3}\sin 3t$ V.

(a) By applying Kirchhoff's Voltage Law, show that the *RLC* circuit can be governed by

$$\frac{d^2i}{dt^2}+2\frac{di}{dt}+10i=37\cos 3t.$$

(4 marks)

(b) Hence, find the general solution for the differential equation in (a).

(12 marks)

(c) Given that $i(0)=0$ and $i'(0)=4$, obtain the particular solution for the the differential equation in (a).

(4 marks)

Q6 (a) Solve the following first-order differential equation.

$$(x^2+2y)dx+(2x+e^y)dy=0$$

(8 marks)

(b) Given the non-homogeneous linear system

$$\begin{aligned} y_1' &= \quad - y_2 + x \\ y_2' &= 3y_1 + 4y_2 - 2-4x \end{aligned}$$

and the general solution of the corresponding homogeneous system is

$$y_C(x)=C_1\begin{bmatrix}1\\-1\end{bmatrix}e^x+C_2\begin{bmatrix}1\\-3\end{bmatrix}e^{3x},$$

where C_1 and C_2 are any constants and $y_C(x)$ is complimentary function.

(i) Find the particular solution, $y_p(x)$ by method of undetermined coefficient.

(ii) Hence, obtain the general solution and particular solution for the above system. Given that $y_1(0)=0$, and $y_2(0)=0$.

(12 marks)

END OF QUESTION

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FORMULAS

Second-order Differential Equation

The roots of characteristic equation and the general solution for differential equation $ay'' + by' + cy = 0$.

Characteristic equation: $am^2 + bm + c = 0$.		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y = Ae^{m_1 x} + Be^{m_2 x}$
2.	Real and equal roots: $m = m_1 = m_2$	$y = (A + Bx)e^{mx}$
3.	Complex roots: $m_1 = \alpha + \beta i$, $m_2 = \alpha - \beta i$	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Laplace Transform

$f(t)$	$F(s)$
a	$\frac{a}{s}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
t^n, $n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$
$e^{at} f(t)$	$F(s-a)$
$t^n f(t)$, $n = 1, 2, 3, \ldots$	$(-1)^n \frac{d^n}{ds^n} F(s)$
$H(t-a)$	$\frac{e^{-as}}{s}$
$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\delta(t-a)$	e^{-as}
$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
y	$Y(s)$
y'	$sY(s) - y(0)$
y''	$s^2Y(s) - sy(0) - y'(0)$

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Fourier Series

Fourier series expansion of periodic function with period 2π	Half Range series
$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$	$a_0 = \frac{2}{L}\int_0^L f(x)\,dx$
$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$	$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx$
$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$	$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx$
$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n\cos nx + \sum_{n=1}^{\infty} b_n\sin nx$	$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n\sin\frac{n\pi x}{L}$

Table of Fourier Transform (Fourier Transform Pairs)

$f(t)$	$F(\omega)$	$f(t)$	$F(\omega)$
$\delta(t)$	1	$\text{sgn}(t)$	$\frac{2}{i\omega}$
$\delta(t-\omega_0)$	$e^{-i\omega_0\omega}$	$H(t)$	$\pi\delta(\omega)+\frac{1}{i\omega}$
1	$2\pi\delta(\omega)$	$e^{-\omega_0 t}H(t)$ for $\omega_0>0$	$\frac{1}{\omega_0+i\omega}$
$e^{i\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	$t^n e^{-\omega_0 t}H(t)$ for $\omega_0>0$	$\frac{n!}{(\omega_0+i\omega)^{n+1}}$
$\sin(\omega_0 t)$	$i\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	$e^{-at}\sin(\omega_0 t)H(t)$ for $a>0$	$\frac{\omega_0}{(a+i\omega)^2+\omega_0^2}$
$\cos(\omega_0 t)$	$\pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$	$e^{-at}\cos(\omega_0 t)H(t)$ for $a>0$	$\frac{a+i\omega}{(a+i\omega)^2+\omega_0^2}$
$\sin(\omega_0 t)H(t)$	$\frac{\pi}{2}i[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$		
$\cos(\omega_0 t)H(t)$	$\frac{\pi}{2}[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]+\frac{i\omega}{\omega_0^2-\omega^2}$		