UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2012/2013

COURSE NAME	:	ENGINEERING MATHEMATICS I
COURSE CODE	:	BWM10103 / BSM1913
PROGRAMME	:	1 BEB / BEC / BED / BEF / BEH 2 BED / BEF / BEH 4 BEE / BFF
EXAMINATION DATE	:	JUNE 2013
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS.

THIS QUESTION PAPER CONSISTS OF **SIX (6)** *PAGES*

Q1 (a) Find

(i) $\lim_{x\to\infty}\sqrt{\dfrac{x^3+7x}{4x^3+5}}$.

(ii) $\lim_{x\to 7}\dfrac{\frac{1}{x}-\frac{1}{7}}{x-7}$.

(iii) $\lim_{x\to a}\dfrac{x^2-a^2}{x-a}$.

(9 marks)

(b) By using L'Hôpital's rule, find

(i) $\lim_{x\to 0}\dfrac{\sin x - x}{\sin x}$.

(ii) $\lim_{x\to 10}\dfrac{x^2-100}{x-10}$.

(8 marks)

(c) Given

$$f(x)=\begin{cases} x-2, & (x<0), \\ x^2, & (x\geq 0). \end{cases}$$

Find

(i) $\lim_{x\to 0^-} f(x)$.

(ii) $\lim_{x\to 0^+} f(x)$.

(iii) $\lim_{x\to 0} f(x)$.

(iv) $f(0)$.

(v) Is the function continuous at $x=0$? Give the reason.

(8 marks)

Q2 (a) Differentiate the following functions with respect to x.

(i) $y = \dfrac{3}{(1-2x)^2}$

(ii) $y = x^{\ln x}$

(iii) $y = \dfrac{\cos x}{1+\sin x}$

(12 marks)

(b) If $3y^2 - 2x^2 = 2xy$, find $\dfrac{dy}{dx}$ in terms of x and y.

(5 marks)

(c) Let $f(x) = (x^2 - 1)^3$.
(i) Find all critical points of $f(x)$.
(ii) Hence, determine whether the critical points is minimum, maximum or inflection point.

(8 marks)

Q3 (a) Find the following integrals

(i) $\int \dfrac{t+1}{t}\, dt.$

(ii) $\int (x-1)e^{x^2-2x+1}\, dx.$

(7 marks)

(b) Evaluate

(i) $\int \dfrac{x^2}{\sqrt[4]{x^3+2}}\, dx.$

(ii) $\int_0^1 \dfrac{e^{3x-1}}{e^{x+2}}\, dx.$

(8 marks)

(c) Show that $\int \dfrac{3x+5}{(x+1)(x-1)^2}\, dx = \dfrac{1}{2}\ln\left|\dfrac{x+1}{x-1}\right| - \dfrac{4}{x-1} + C.$

(10 marks)

Q4 (a) Find $f'(x)$ for each of the following functions.

(i) $f(x) = \tan^{-1}\left(\dfrac{1+4x}{1-4x}\right)$

(ii) $f(x) = \sin^{-1}(\ln x)$

(10 marks)

(b) Find area of the surface that is generated by revolving the curve $y = \sqrt[3]{3x}$ between $y = -1$ and $y = 0$ about the y-axis.

(10 marks)

(c) Find the curvature if $x = \cos t$ and $y = \ln 2t$ at $t = \pi$.

(5 marks)

- END OF QUESTION -

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Formulae

Indefinite Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x}\, dx = \ln|x| + C$$

$$\int \cos x\, dx = \sin x + C$$

$$\int \sin x\, dx = -\cos x + C$$

$$\int \sec^2 x\, dx = \tan x + C$$

$$\int \csc^2 x\, dx = -\cot x + C$$

$$\int \sec x \tan x\, dx = \sec x + C$$

$$\int \csc x \cot x\, dx = -\csc x + C$$

$$\int e^x dx = e^x + C$$

$$\int \cosh x\, dx = \sinh x + C$$

$$\int \sinh x\, dx = \cosh x + C$$

$$\int \text{sech}^2 x\, dx = \tanh x + C$$

$$\int \text{csch}^2 x\, dx = -\coth x + C$$

$$\int \text{sech}\, x \tanh x\, dx = -\text{sech}\, x + C$$

$$\int \text{csch}\, x \coth x\, dx = -\text{csch}\, x + C$$

Integration of Inverse Functions

$$\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x + C, \quad |x| < 1$$

$$\int \frac{-1}{\sqrt{1-x^2}}\, dx = \cos^{-1} x + C, \quad |x| < 1$$

$$\int \frac{1}{1+x^2}\, dx = \tan^{-1} x + C$$

$$\int \frac{-1}{1+x^2}\, dx = \cot^{-1} x + C$$

$$\int \frac{1}{|x|\sqrt{x^2-1}}\, dx = \sec^{-1} x + C, \quad |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{x^2-1}}\, dx = \csc^{-1} x + C, \quad |x| > 1$$

$$\int \frac{1}{\sqrt{x^2+1}}\, dx = \sinh^{-1} x + C$$

$$\int \frac{1}{\sqrt{x^2-1}}\, dx = \cosh^{-1} x + C, \quad |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{1-x^2}}\, dx = \text{sech}^{-1} |x| + C, \quad 0 < x < 1$$

$$\int \frac{-1}{|x|\sqrt{1+x^2}}\, dx = \text{csch}^{-1} |x| + C, \quad x \neq 0$$

$$\int \frac{1}{1-x^2}\, dx = \begin{cases} \tanh^{-1} x + C, & |x| < 1 \\ \coth^{-1} x + C, & |x| > 1 \end{cases}$$

TAYLOR AND MACLAURIN SERIES

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

TRIGONOMETRIC SUBSTITUTION

Expression	***Trigonometry***	***Hyperbolic***
$\sqrt{x^2+k^2}$	$x = k\tan\theta$	$x = k\sinh\theta$
$\sqrt{x^2-k^2}$	$x = k\sec\theta$	$x = k\cosh\theta$
$\sqrt{k^2-x^2}$	$x = k\sin\theta$	$x = k\tanh\theta$

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Formulae

TRIGONOMETRIC SUBSTITUTION

$t = \tan \frac{1}{2}x$		$t = \tan x$	
$\sin x = \dfrac{2t}{1+t^2}$ $\tan x = \dfrac{2t}{1-t^2}$	$\cos x = \dfrac{1-t^2}{1+t^2}$ $dx = \dfrac{2dt}{1+t^2}$	$\sin 2x = \dfrac{2t}{1+t^2}$ $\tan 2x = \dfrac{2t}{1-t^2}$	$\cos 2x = \dfrac{1-t^2}{1+t^2}$ $dx = \dfrac{dt}{1+t^2}$

IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

Trigonometric Functions

$\cos^2 x + \sin^2 x = 1$

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$
$= 2\cos^2 x - 1$
$= 1 - 2\sin^2 x$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

$\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$

$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

$2\sin ax \cos bx = \sin(a+b)x + \sin(a-b)x$

$2\sin ax \sin bx = \cos(a-b)x - \cos(a+b)x$

$2\cos ax \cos bx = \cos(a-b)x + \cos(a+b)x$

Hyperbolic Functions

$\sinh x = \dfrac{e^x - e^{-x}}{2}$

$\cosh x = \dfrac{e^x + e^{-x}}{2}$

$\cosh^2 x - \sinh^2 x = 1$

$\sinh 2x = 2\sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$
$= 2\cosh^2 x - 1$
$= 1 + 2\sinh^2 x$

$1 - \tanh^2 x = \text{sech}^2 x$

$\coth^2 x - 1 = \text{csch}^2 x$

$\tanh 2x = \dfrac{2\tanh x}{1+\tanh^2 x}$

$\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$

$\sinh(x \pm y) = \sinh x \cosh y \pm \sinh y \cosh x$

$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$

CURVATURE, ARC LENGTH AND SURFACE AREA OF REVOLUTION

$$\kappa = \frac{\left|\dfrac{d^2y}{dx^2}\right|}{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}$$

$$\kappa = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{\left[\dot{x}^2 + \dot{y}^2\right]^{3/2}}$$

$$L = \int_{x_1}^{x_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$L = \int_{y_1}^{y_2} \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$S = 2\pi \int_{x_1}^{x_2} f(x)\sqrt{1+\left(\frac{d}{dx}[f(x)]\right)^2}\, dx$$

$$S = 2\pi \int_{y_1}^{y_2} g(y)\sqrt{1+\left(\frac{d}{dy}[g(y)]\right)^2}\, dy$$