UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESI 2012/2013

COURSE NAME	:	LINEAR ALGEBRA
COURSE CODE	:	BWA10303
PROGRAMME	:	1 BWA/1 BWQ
EXAMINATION DATE	:	DECEMBER 2012/JANUARY 2013
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **FOUR (4)** PAGES

Q1 a) Let $A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$. Find all the eigenvalues of A and their corresponding eigenvectors.

b) Show that if B is a matrix with three distinct eigenvalues α, β, γ and corresponding eigenvectors u, v, w, then u, v, w are linearly independent. Hence, deduce that the eigenvectors found in i) are a linearly independent.

c) Find a nonsingular matrix P such that $P^{-1}AP$ is diagonalizable.

d) Using the result in ii), find A^5.

e) Determine whether or not A is similar to $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

(30 marks)

Q2 Let

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 3 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

State which of the following expression do not define a matrix and evaluate the others:

$$AB, \quad BC, \quad AC, \quad (AB)^T, \quad B^TC, \quad B^TA^T, \quad 2A+B$$

(10 marks)

Q3 Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. Prove that $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$.

(10 marks)

Q4 a) Given that $|A| = -2$ find $|A^{-1}|$ and $|A^{100}|$.

b) Given that $|A| = 2$, and $|B| = 5$, find $|AB^T|$.

c) Let A be a square matrix of order n. Form the relation $AA^* = |A| I_n$ show that if A is invertible, then $|A^*| = |A|^{n-1}$. Here, A^* is adj(A).

d) Let

$$A = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & p & q \end{bmatrix} \text{ and } |A| = 2.$$

Find the following determinants:

(i) $A_1 = \begin{bmatrix} \lambda a & \lambda b & \lambda c & \lambda d \\ \lambda e & \lambda f & \lambda g & \lambda h \\ \lambda i & \lambda j & \lambda k & \lambda l \\ \lambda m & \lambda n & \lambda p & \lambda q \end{bmatrix}$ (ii) $A_2 = \begin{bmatrix} a & b & c & d \\ i & j & k & l \\ e & f & g & h \\ m & n & p & q \end{bmatrix}$

(iii) $A_3 = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m+2a-5i & n+2b-5j & p+2c-5k & q+2d-5l \end{bmatrix}$

(iv) $A_4 = \begin{bmatrix} a & b & a & c & d \\ e & f & b & g & h \\ 0 & 0 & 1 & 0 & 0 \\ i & j & c & k & l \\ m & n & d & p & q \end{bmatrix}$

(15 marks)

Q5 a) Define "Row Echelon Matrix" and "Reduced Row Echelon Matrix".

b) Give an example of a 4×5 reduce row echelon matrix with a zero row and leading entries in columns 1,3 and 5.

c) Let $A = \begin{bmatrix} 2 & 7 & 3 & 5 \\ 0 & 3 & 6 & 6 \\ 1 & 1 & 1 & 2 \end{bmatrix}$. Reduce A to an r.e. matrix by elementary row operation and find the rank of A.

d) Find all possible values of rank(B)

$$B = \begin{bmatrix} 1 & 2 & a \\ -2 & 4a & 2 \\ a & -2 & 1 \end{bmatrix}$$

(25 marks)

Q6 a) Give the definition of idempotent matrix. Thus, if $B = \beta \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ is idempotent, find β.

b) For what values of $\alpha, \beta, \sigma, \delta, \varepsilon$ and θ is the matrix

$$A = \begin{bmatrix} \alpha & -2 & \beta \\ \delta & \theta & \varepsilon \\ 5 & 3 & \sigma \end{bmatrix}$$

i) symmetric

ii) skew-symmetric

(10 marks)