UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2011/2012

COURSE NAME	:	ENGINEERING MATHEMATICS IV
COURSE CODE	:	BWM 30603/BSM 3913
PROGRAMME	:	1 BEE 2 BDD/BEE/BFF 3 BDD/BEE/BFF 4 BDD/ BFF
EXAMINATION DATE	:	JANUARY 2012
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS IN **PART A** AND **TWO (2)** QUESTIONS IN **PART B.** ALL CALCULATIONS AND ANSWERS MUST BE IN THREE (3) DECIMAL PLACES.

THIS EXAMINATION PAPER CONSISTS OF SEVEN (7) PAGES

PART A

Q1 (a) Consider the heat conduction equation

$$\frac{\partial}{\partial t}T(x,t)=\alpha\frac{\partial^2}{\partial x^2}T(x,t),\quad 0<x<10,\ t>0,$$

where α is thermal diffusity $=10$, since $\alpha=c^2$.

Given the boundary conditions,

$$T(0,t)=0,\ \ T(10,t)=100$$

and initial condition,

$$T(x,0)=x^2.$$

By using explicit finite-difference method, find $T(x,0.055)$ and $T(x,0.11)$ with 5 grid intervals on the x coordinate.

(10 marks)

(b) Let $y(x,t)$ denotes displacement of a vibrating string. If T is the tension in the string, ω is the weight per unit length, and g is acceleration due to gravity, then y satisfies the equation

$$\frac{\partial^2 y}{\partial t^2}=\frac{Tg}{\omega}\frac{\partial^2 y}{\partial x^2},\quad 0<x<2,\ t>0.$$

Suppose a particular string is 2 m long and is fixed at both ends. By taking $T=1.5\text{N}$, $\omega=0.01$ kg/m and $g=10$ m/s^2, use the finite-difference method to solve for y up to level 2 only. The initial conditions are

$$y(x,0)=\begin{cases}\dfrac{x}{2}, & 0\le x\le 1\\[2mm] \dfrac{2-x}{2}, & 1\le x\le 2\end{cases}\qquad \text{and}\qquad \frac{\partial y}{\partial t}(x,0)=x(x-2).$$

Performed all calculations with $\Delta x=0.5$ m and $\Delta t=0.01$ s.

(15 marks)

Q2 The steady state temperature distribution of heated rod follows the one-dimensional form of Poisson's equation

$$\frac{d^2T}{dx^2}+Q(x)=0.$$

Solve the above equation for a 6 cm rod with boundary conditions of $T(0,t)=10$ and $T(6,t)=50$ and a uniform heat source $Q(x)=40$ with 3 equal-size elements of length by using finite-element method with linear approximation.

(25 marks)

PART B

Q3 (a) Given $f(x) = 7e^{-x}\sin x - 1$ for $-1 \le x \le 1$.

(i) Find the root of $f(x)$ by using Secant method (iterate until $|f(x_i)| < \varepsilon = 0.005$).

(ii) If the exact solution of $f(x)$ is 0.118, find the absolute error.

(8 marks)

(b) A mixture company has three sizes of packs of nuts. The *Large* size contains 2 kg of walnuts, 2 kg of peanuts and 1 kg of cashews. The *Mammoth* size contains 3 kg of walnuts, 6 kg of peanuts and 2 kg of cashews. The *Giant* size contains 1 kg of walnuts, 4 kg of peanuts and 2 kg of cashews. Suppose that the company receives an order for 14 kg of walnuts, 26 kg of peanuts and 12 kg of cashews.

(i) By taking *a, b* and *c* represent *Large*, *Mammoth* and *Giant* size, obtain the system of linear equations for this company.

(ii) By using Gauss elimination method, determine how can this company fill this order with the given sizes of packs.

(iii) Suppose that the mixture company above is planning to expand their mixing productions. Their planning can be summarized as **Table Q3(b)** below:

	Walnuts	Peanuts	Cashews	Almond
Large	10	4	2	2
Mammoth	1	5	11	1
Giant	4	14	2	1
Small	3	3	2	9

Table Q3(b)

If the company receives a new order for 13 kg of walnuts, 34 kg of peanuts, 15 kg of cashews and 28 kg of Almond, determine the possible solutions of the system by using Gauss-Seidel iteration method.

(17 marks)

Q4 (a) A certain lab experiment produced the following data (**Table Q4(a)**):

x	$y(x)$
0	-100
20	280
40	1460
60	3440
80	6220

Table Q4(a)

Predict $y(x)$, when $x = 70$ by using
(i) Lagrange polynomial interpolation and
(ii) Newton divided-difference interpolation.

(16 marks)

(b) Evaluate $\int_0^1 e^{x^2} \sin x \; dx$ by using 3-point Gauss Quadrature.

(9 marks)

Q5 (a) Given the matrix

$$A = \begin{pmatrix} 3 & 4 & 1 \\ 4 & 3 & 0 \\ 1 & 4 & 3 \end{pmatrix}$$

(i) By using the power method, compute the dominant eigenvalue, λ_{Largest} of A and its corresponding eigenvector v_1.

(ii) Then, find smallest eigenvalue, $\lambda_{\text{Smallest}}$ of A by using the shifted power method.

For those both calculations, use initial eigenvector, $v^{(0)} = (1 \quad 1 \quad 1)^T$ and stop the iteration when $|m_{k+1} - m_k| < \varepsilon = 0.005$.

(8 marks)

(b) (i) The differential equation

$$\frac{dp}{dt} = rb(1-p), \quad p(0) = p_0,$$

is a model for studying the proportion, $p(t)$ of nonconformist in a society. The birth rate is b and the rate at which offspring would become nonconformist when at least one of their parents was a conformist is r. If $p(0) = 0.01$, $b = 0.02$ and $r = 0.1$, approximate $p(3)$ by using second-order Taylor series method and modified Euler's method. Assume that $h = \Delta t = 1$ year.

(ii) Solve the following boundary value problem

$$y'' + 4y' + 4y = e^{-t}, \ y(0) = 0, y(1) = 0.$$

Consider $h = \Delta t = 0.25$.

(17 marks)

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FORMULAS

Nonlinear equations

Secant method : $x_{i+2} = \dfrac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}$

System of linear equations

Gauss-Seidel iteration method:

$$x_i^{(k+1)} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}}{a_{ii}}, i = 1,2,\ldots,n$$

Interpolation

Lagrange polynomial: $P_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i), i = 0,1,2,\ldots,n$ where $L_i(x) = \prod_{\substack{j=0 \\ j\neq i}}^{n} \dfrac{(x - x_j)}{(x_i - x_j)}$

Newton divided difference:

$$P_n(x) = f_0^{[0]} + f_0^{[1]}(x - x_0) + f_0^{[2]}(x - x_0)(x - x_1) + \ldots + f_0^{[n]}(x - x_0)(x - x_1)\ldots(x - x_{n-1})$$

Numerical differentiation and integration

Integration:

Gauss quadrature:

For $\int_a^b f(x)dx$, $\quad x = \dfrac{(b-a)t + (b+a)}{2}$

3-points: $\int_{-1}^{1} f(x)dx \approx \dfrac{5}{9} g\left(-\sqrt{\dfrac{3}{5}}\right) + \dfrac{8}{9} g(0) + \dfrac{5}{9} g\left(\sqrt{\dfrac{3}{5}}\right)$

Eigenvalue

Power Method: $v^{(k+1)} = \dfrac{1}{m_{k+1}} A v^{(k)}, \quad k = 0,1,2,\ldots$

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Ordinary differential equations

Initial value problems:

Modified Euler's method: $y_{i+1} = y_i + \frac{1}{2}k_1 + \frac{1}{2}k_2$

where $k_1 = hf(x_i, y_i)$, $k_2 = hf(x_i + h,\ y_i + k_1)$

Second order Taylor series method: $y(x_{i+1}) = y(x_i) + hy'(x_i) + \frac{h^2}{2!}y''(x_i)$

Boundary value problems:

Finite difference method:

$$y_i' \approx \frac{y_{i+1} - y_{i-1}}{2h}, \qquad y_i'' \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

Partial differential equation

Heat Equation: Finite difference method

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \qquad \frac{u_{i,j+1} - u_{i,j}}{k} = c^2 \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2}$$

Wave equation: Finite difference method

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \qquad \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

Finite element method

$KT = F_b - F_l,$

where $K_{ij} = A_{ij} = \int_p^q a \frac{dN_i}{dx}\frac{dN_j}{dx}dx$ is stiffness matrix,

$$T = T_i \quad , \quad F_b = \left[N_i a \frac{dT}{dx}\right]_p^q \quad , \quad F_l = \int_p^q N_i f(x)dx$$