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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2011/2012

COURSE NAME	:	ENGINEERING MATHEMATICS I
COURSE CODE	:	BWM 10103 / BSM 1913
PROGRAMME	:	1 BFF / 2 BFF / 3 BFF / 4 BFF 1 BEE / 2 BEE / 3 BEE / 4 BEE 1 BDD / 2 BDD
EXAMINATION DATE	:	JANUARY 2012
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS EXAMINATION PAPER CONSISTS OF SIX (6) PAGES

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Q1 (a) A sheet of metal with 12 meters by 10 meters is to be used to make an open box. Squares of equal sides x are cut out of each corner then the sides are folded to make the box, see **Figure Q1**.

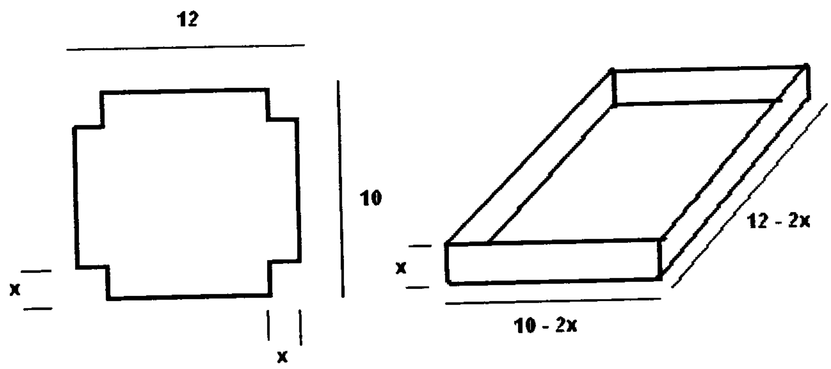

Figure Q1: Making a box

(i) Formulate the volume V of the box.
(ii) Obtain the critical points when $dV/dx = 0$.
(iii) Determine the value of x that makes the volume maximum.

(10 marks)

(b) Applying $\cos u = x$, show that

$$\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}}, \text{ for } |x| < 1.$$

Then, find the derivative of $y = \cos^{-1} 5x$.

(10 marks)

Q2 (a) Find the surface area generated by a line $y = 4x + 2$ from $y = 0$ to $y = \frac{1}{2}$ is rotated $360°$ about y- axis.

(5 marks)

(b) Find the arc length of a parametric curve $x = 2\cos^2 t$ and $y = 2\sin^2 t$ over the interval $t = 0$ to $t = \frac{\pi}{2}$.

(5 marks)

(c) Given $y^2 - 4x^2 = 9$

(i) Find the curvature κ of the given curve at $x = 2$.
(ii) Find the radius of curvature ρ of the given curve at $x = 1$.

(10 marks)

Q3 (a) Evaluate the following limits (if exist)

(i) $\lim_{x\to\infty}\sqrt{\dfrac{x^3+7x}{4x^3+5}}$.

(ii) $\lim_{x\to\infty}\left(x-\sqrt{x^2+7}\right)$.

(6 marks)

(b) Evaluate $\lim_{x\to 1}\dfrac{(x-1)\sin(x-1)}{1-\cos(x-1)}$, using $\lim_{x\to 0}\dfrac{\sin x}{x}=1$.

(6 marks)

(c) Determine all values of the constant A and B so that the following function is continuous for all values of x.

$$f(x)=\begin{cases} Ax-B, & \text{if } x\le -1, \\ 2x^2+3Ax+B, & \text{if } -1<x\le 1, \\ 4, & \text{if } x>1. \end{cases}$$

(8 marks)

Q4 (a) Evaluate

(i) $\int \dfrac{e^x}{\sqrt{e^{2x}+16}}\,dx$.

(ii) $\int_0^{0.5}\cos^{-1}x\,dx$.

(6 marks)

(b) Given $x=\dfrac{t-3}{t}$ and $y=\dfrac{t^2+5}{t}$, find $\dfrac{dy}{dx}$ when $x=1$.

(6 marks)

(c) A rocket rising vertically is tracked by a radar station that is on the ground 5 meters from the launch pad. How fast is the rocket rising when it is 4 meters high and its distance from the radar station is increasing at a rate of 2000 meters / hour?

(8 marks)

Q5 (a) Consider the power series

$$\sum_{n=o}^{\infty} \frac{(x-6)^n}{n+1}$$

Find the radius and interval of convergence of the given power series.

(12 marks)

(b) Show that the Maclaurin series for $f(x) = e^{-x}$ is

$$1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} \; \ldots\ldots.$$

Then, evaluate $\int_0^1 e^{x^2} dx$.

(8 marks)

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Formulae

Indefinite Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x}\, dx = \ln|x| + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \csc x \cot x \, dx = -\csc x + C$$

$$\int e^x dx = e^x + C$$

$$\int \cosh x \, dx = \sinh x + C$$

$$\int \sinh x \, dx = \cosh x + C$$

$$\int \text{sech}^2 x \, dx = \tanh x + C$$

$$\int \text{csch}^2 x \, dx = -\coth x + C$$

$$\int \text{sech}\, x \tanh x \, dx = -\text{sech}\, x + C$$

$$\int \text{csch}\, x \coth x \, dx = -\text{csch}\, x + C$$

Integration of Inverse Functions

$$\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x + C, \ |x| < 1$$

$$\int \frac{-1}{\sqrt{1-x^2}}\, dx = \cos^{-1} x + C, \ |x| < 1$$

$$\int \frac{1}{1+x^2}\, dx = \tan^{-1} x + C$$

$$\int \frac{-1}{1+x^2}\, dx = \cot^{-1} x + C$$

$$\int \frac{1}{|x|\sqrt{x^2-1}}\, dx = \sec^{-1} x + C, \ |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{x^2-1}}\, dx = \csc^{-1} x + C, \ |x| > 1$$

$$\int \frac{1}{\sqrt{x^2+1}}\, dx = \sinh^{-1} x + C$$

$$\int \frac{1}{\sqrt{x^2-1}}\, dx = \cosh^{-1} x + C, \ |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{1-x^2}}\, dx = \text{sech}^{-1} |x| + C, \ 0 < x < 1$$

$$\int \frac{-1}{|x|\sqrt{1+x^2}}\, dx = \text{csch}^{-1} |x| + C, \ x \neq 0$$

$$\int \frac{1}{1-x^2}\, dx = \begin{cases} \tanh^{-1} x + C, & |x| < 1 \\ \coth^{-1} x + C, & |x| > 1 \end{cases}$$

TAYLOR AND MACLAURIN SERIES

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

TRIGONOMETRIC SUBSTITUTION

Expression	*Trigonometry*	*Hyperbolic*
$\sqrt{x^2+k^2}$	$x = k\tan\theta$	$x = k\sinh\theta$
$\sqrt{x^2-k^2}$	$x = k\sec\theta$	$x = k\cosh\theta$
$\sqrt{k^2-x^2}$	$x = k\sin\theta$	$x = k\tanh\theta$

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Formulae

TRIGONOMETRIC SUBSTITUTION

$t = \tan\frac{1}{2}x$		$t = \tan x$	
$\sin x = \dfrac{2t}{1+t^2}$	$\cos x = \dfrac{1-t^2}{1+t^2}$	$\sin 2x = \dfrac{2t}{1+t^2}$	$\cos 2x = \dfrac{1-t^2}{1+t^2}$
$\tan x = \dfrac{2t}{1-t^2}$	$dx = \dfrac{2dt}{1+t^2}$	$\tan 2x = \dfrac{2t}{1-t^2}$	$dx = \dfrac{dt}{1+t^2}$

IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

Trigonometric Functions

$\cos^2 x + \sin^2 x = 1$

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$= 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

$\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

$\sin(x \pm y) = \sin x\cos y \pm \sin y\cos x$

$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$

$2\sin ax\cos bx = \sin(a+b)x + \sin(a-b)x$

$2\sin ax\sin bx = \cos(a-b)x - \cos(a+b)x$

$2\cos ax\cos bx = \cos(a-b)x + \cos(a+b)x$

Hyperbolic Functions

$\sinh x = \dfrac{e^x - e^{-x}}{2}$

$\cosh x = \dfrac{e^x + e^{-x}}{2}$

$\cosh^2 x - \sinh^2 x = 1$

$\sinh 2x = 2\sinh x\cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$= 2\cosh^2 x - 1$

$= 1 + 2\sinh^2 x$

$1 - \tanh^2 x = \text{sech}^2 x$

$\coth^2 x - 1 = \text{csch}^2 x$

$\tanh 2x = \dfrac{2\tanh x}{1+\tanh^2 x}$

$\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x\tanh y}$

$\sinh(x \pm y) = \sinh x\cosh y \pm \sinh y\cosh x$

$\cosh(x \pm y) = \cosh x\cosh y \pm \sinh x\sinh y$

CURVATURE, ARC LENGTH AND SURFACE AREA OF REVOLUTION

$$\kappa = \frac{\left|\dfrac{d^2y}{dx^2}\right|}{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}$$

$$\kappa = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{\left[\dot{x}^2 + \dot{y}^2\right]^{3/2}}$$

$$L = \int_{x_1}^{x_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$$L = \int_{y_1}^{y_2} \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$S = 2\pi\int_{x_1}^{x_2} f(x)\sqrt{1+\left(\frac{d}{dx}[f(x)]\right)^2}\,dx$$

$$S = 2\pi\int_{y_1}^{y_2} g(y)\sqrt{1+\left(\frac{d}{dy}[g(y)]\right)^2}\,dy$$