UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2011/2012

COURSE NAME	:	ENGINEERING MATHEMATICS IV
COURSE CODE	:	BWM 30603/BSM 3913
PROGRAMME	:	2 BDD/BFF 3 BDD/BFF
EXAMINATION DATE	:	JUNE 2012
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS IN **PART A** AND **TWO (2)** QUESTIONS IN **PART B.** ALL CALCULATIONS AND ANSWERS MUST BE IN THREE (3) DECIMAL PLACES.

THIS EXAMINATION PAPER CONSISTS OF **SEVEN (7)** PAGES

PART A

Q1 (a) Given the heat equation

$$\frac{\partial u(x,t)}{\partial t} = 2\frac{\partial^2 u(x,t)}{\partial x^2}, \quad 0 \le x \le 2, \quad t > 0,$$

with the boundary conditions

$$u(0,t) = u(2,t) = 0,$$

and the initial condition

$$u(x,0) = \sin(\pi x).$$

Find $u(x, 0.3)$ by using Implicit Crank-Nicolson method with $\Delta x = h = 0.5$ and $\Delta t = k = 0.3$.

(10 marks)

(b) The longitudinal vibration of a bar with the length of l m is governed by

$$c^2 \frac{\partial^2 \phi}{\partial x^2} = \frac{\partial^2 \phi}{\partial t^2}$$

with $c = \sqrt{\frac{E}{\rho}}$, where $\phi = \phi\ (x,t)$ is the axial displacement, E is Young's modulus and ρ is the mass density of the bar. The boundary conditions and the initial conditions are given as follows,

$$\phi\ (0,t) = \phi\ (l,t) = 0 \text{ for } 0 \le t \le 0.04$$

$$\phi(x,0) = 0 \text{ and } \frac{\partial \phi(x,0)}{\partial t} = x \text{ for } 0 \le x \le 20.$$

Determine the variation of the axial displacement of the bar by using finite-difference method with the following data:
$E = 30 \times 10^6$, $\rho = 0.264$, $l = 20$ m, $\Delta x = h = 5$ and $\Delta t = k = 0.02$.

(15 marks)

Q2

Figure Q2

Consider a fin of length 4 unit has four nodes and three elements, as shown in **Figure Q2**. The heat flow equation is given by

$$\frac{d}{dx}\left(A(x)k(x)\frac{dT(x)}{dx}\right)+Q(x)=0, \quad \text{for} \quad 0 \le x \le 4,$$

with $A(x)$ is the cross-sectional area, $k(x)$ is the thermal conductivity, $T(x)$ is the temperature at length x and $Q(x)$ is the heat supply per unit time and per unit length. Find the temperature at each nodal point, T_2, T_3 and T_4, if $A(x)$ is 30 unit, $k(x)$ is 10 unit and $Q(x)$ is 10 unit. Let the temperature at $x = 0$ is 0 unit and the heat flux, $-k\left.\frac{dT}{dx}\right|_{x=4} = 10$ unit.

(25 marks)

PART B

Q3 (a) Table below shows a set of discrete data.

x	1.1	1.3	1.5	1.7
$y(x)$	0.907	?	1.355	1.999

Find the missing value of $y(x)$ by using Lagrange interpolation method.

(10 marks)

(b) Use the result in Q4(a) to find $y'(1.3)$ and $y''(1.3)$ by using any appropriate difference formulas with the suitable step size of h.

(5 marks)

(c) Find the approximate value of $\int_0^3 3e^{\left(\frac{-x^2}{2}\right)}dx$ by using

(i) Simpson's $\frac{3}{8}$ rule with $n = 6$, and

(ii) 3-points Gaussian Quadrature.

(10 marks)

Q4 (a) Find the largest eigenvalue and its corresponding eigenvector for the matrix A below by using power method.

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -2 & 3 & -2 \\ 0 & -1 & 2 \end{pmatrix}$$

Use the initial guess for eigenvector $v^{(0)} = (1 \quad 1 \quad 1)^T$. Calculate until $|m_{k+1} - m_k| < 0.005$.

(10 marks)

(b) Approximate the solution at $y(1.2)$ for the initial-value problem $y' = \dfrac{(x+y)^2}{2xy + x^2 - 1}$ with $y(1) = 1$, using the fourth order Runge-Kutta (RK4) method with the step size $h = 0.2$.

(7 marks)

(c) Given the boundary-value problem of

$$y'' + xy' - 5(1+x)y = x^3$$

for $0 \le x \le 2$ with the following boundary conditions

$$4y(0) + y'(0) = 2, \quad 3y(2) + 2y'(2) = 5.$$

Derive the system of linear equation by using finite-difference method with grid size $h = \Delta x = 0.5$. **DO NOT** solve the obtained system.

(8 marks)

Q5 (a) Locate the positive root of the nonlinear equation $x^2 - 4\sin(x) = 0$ by using Intermediate value theorem. Hence, solve it by using Bisection method.

(10 marks)

(b) Given the system of linear equations:

$$\begin{pmatrix} 2 & -4 & -1 \\ 2 & -2 & -5 \\ -3 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 8 \\ 9 \end{pmatrix}.$$

Solve it by

(i) Doolittle method, and

(ii) Gauss-Seidel iteration method.

(15 marks)

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FORMULAE

Bisection method: $c_i = \dfrac{a_i + b_i}{2}$

Doolittle method

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ l_{21} & 1 & 0 & \cdots & 0 \\ l_{31} & l_{32} & 1 & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 \\ l_{n1} & l_{n2} & l_{n3} & \cdots & 1 \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ 0 & u_{22} & u_{23} & \cdots & u_{2n} \\ 0 & 0 & u_{33} & \cdots & u_{3n} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & u_{nn} \end{pmatrix}$$

Gauss-Seidel iteration method: $x_i^{(k+1)} = \dfrac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}}{a_{ii}}, i = 1, 2, \ldots, n$

Lagrange polynomial

$P_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i), i = 0, 1, 2, \ldots, n$ where $L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \dfrac{(x - x_j)}{(x_i - x_j)}$

Numerical Differentiation:

First derivatives:

2-point forward difference: $f'(x) \approx \dfrac{f(x+h) - f(x)}{h}$

2-point backward difference: $f'(x) \approx \dfrac{f(x) - f(x-h)}{h}$

3-point forward difference: $f'(x) \approx \dfrac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h}$

3-point backward difference: $f'(x) \approx \dfrac{3f(x) - 4f(x-h) + f(x-2h)}{2h}$

3-point central difference: $f'(x) \approx \dfrac{f(x+h) - f(x-h)}{2h}$

5-point difference: $f'(x) \approx \dfrac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}$

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Second derivatives:

3-point central difference: $f''(x) \approx \dfrac{f(x+h) - 2f(x) + f(x-h)}{h^2}$

5-point difference: $f''(x) \approx \dfrac{-f(x+2h) + 16f(x+h) - 30f(x) + 16f(x-h) - f(x-2h)}{12h^2}$

Numerical Integration:

Simpson's $\frac{3}{8}$ rule

$$\int_a^b f(x)dx \approx \frac{3}{8}h\left[f_0 + f_n + 3(f_1 + f_2 + f_4 + f_5 + \ldots + f_{n-2} + f_{n-1}) + 2(f_3 + f_6 + \ldots + f_{n-6} + f_{n-3})\right]$$

Gauss quadrature

For $\int_a^b f(x)dx$, $\quad x = \dfrac{(b-a)t + (b+a)}{2}$

3-points: $\displaystyle\int_{-1}^{1} g(t)\,dt \approx \frac{5}{9}g\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9}g(0) + \frac{5}{9}g\left(\sqrt{\frac{5}{5}}\right)$

Power Method $v^{(k+1)} = \dfrac{1}{m_{k+1}} A v^{(k)}, \quad k = 0, 1, 2, \ldots$

Initial value problems

Classical 4th order Runge-Kutta method.

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where $k_1 = hf(x_i, y_i)$ $\qquad k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$ $\qquad k_4 = hf(x_i + h, y_i + k_3)$

Boundary value problems:

Finite difference method

$$y_i' \approx \frac{y_{i+1} - y_{i-1}}{2h}, \qquad y_i'' \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

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Heat Equation: Implicit Crank-Nicolson method

$$\left(\frac{\partial u}{\partial t}\right)_{i,j+\frac{1}{2}} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j+\frac{1}{2}}$$

$$\frac{u_{i,j+1} - u_{i,j}}{k} = \frac{c^2}{2}\left(\frac{u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1}}{h^2} + \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}\right)$$

Wave equation: Finite difference method

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j}$$

$$\frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

$$\frac{\partial u(x,0)}{\partial t} = \frac{u_{i,j+1} - u_{i,j-1}}{2k} = g(x_i)$$

Finite element method:

$$KT = F_b - F_l$$

where $K_{ij} = \int_p^q A(x)k(x)\frac{dN_i}{dx}\frac{dN_j}{dx}dx$ is stiffness matrix,

$$T = T_i$$

$$F_b = \left[N_i A(x)k(x)\frac{dT}{dx}\right]_p^q$$

$$F_l = -\int_p^q N_i Q(x)dx$$