UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2011/2012

COURSE NAME	:	ENGINEERING MATHEMATICS II E
COURSE CODE	:	BWM 10303 / BSM 1933
PROGRAMME	:	1 BEF / BEU / BEE
EXAMINATION DATE	:	JUNE 2012
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS EXAMINATION PAPER CONSISTS OF **SEVEN (7)** PAGES

PART A

Q1 (a) Show that the equation

$$(xy + y^2 + y)dx + (x + 2y)dy = 0$$

is not exact. By using the integrating factor $R(x) = e^x$, solve the equation.

(12 marks)

(b) A simple electrical circuit consists of a constant resistance R (in ohms), constant inductance L (in henrys) and an electromotive force $E(t)$ (in volts). According to Kirchhoff's second law, the current i (in amperes) in the circuit satisfies the equation

$$L\frac{di}{dt} + Ri = E(t).$$

Solve the differential equation with the following conditions, $E(t) = E_0$ is a constant and $i = i_0$ when $t = 0$. Describe the current i when $i \to \infty$.

(8 marks)

Q2 (a) Find the general solution of second-order differential equation by method of undetermined coefficients

$$y'' - 3y' + 2y = 6e^{5x},$$

which satisfies the conditions $y = 1$ and $\frac{dy}{dx} = 1$ when $x = 0$.

(10 marks)

(b) Find the general solution of the differential equation by variation of parameters

$$y'' + 2y' + 5y = e^{-x} \sin 2x.$$

(10 marks)

PART B

Q3 (a)

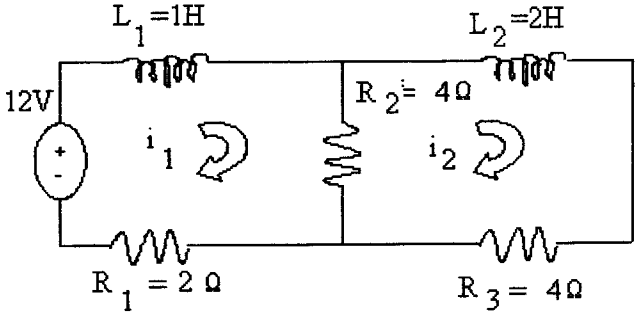


Figure Q3

Refer to the circuit network in Figure Q3 above, show that a model for the current $i_1(t)$ and $i_2(t)$ is given by

$$\begin{pmatrix} i_1' \\ i_2' \end{pmatrix} = \begin{pmatrix} -6 & 4 \\ 2 & -4 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} + \begin{pmatrix} 12 \\ 0 \end{pmatrix}$$

Hence, find the general solution of homogeneous for $i_1(t)$ and $i_2(t)$ in above circuit.

(10 marks)

(b) Use a power series to solve the differential equation

$$y'' + xy' + y = 0.$$

(10 marks)

Q4 (a) By using convolution theorem, show that

$$L^{-1}\left\{\frac{1}{(s^2+b^2)^2}\right\}=\frac{1}{2b^3}(\sin bt-bt\cos bt).$$

Hence solve the initial value problem

$$y''-4y=t\cos 2t$$

given that y and y'' are both zero when $t=0$.

(10 marks)

(b) The RC – circuit in the Figure Q4(b)(i), consists of a resistor R and a capacitor C, connected in series together with a voltage source, $E(t)$ with $R=2.5\,\Omega$, $C=0.08\,F$, $q(0)=0$, $E(t)$ is as given in the Figure Q4(b)(ii).

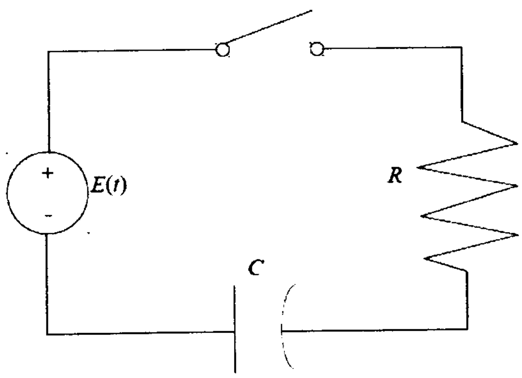
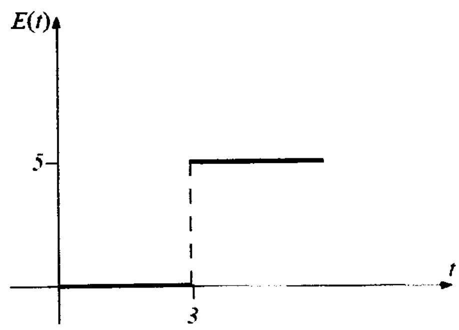


Figure Q4(b)(i)



FigureQ4(b)(ii)

By Kirchhoff's Voltage Law, show that the governing equation for this circuit is

$$2.5\frac{dq(t)}{dt}+12.5q(t)=5H(t-3).$$

Then, use the Laplace transform to find the charge $q(t)$ on the capacitor.

(10 marks)

Q5 (a) A periodic function $f(x)$ is defined as

$$f(x)=\begin{cases}-\cos x & -\pi<x<0,\\ \cos x, & 0\le x\le \pi,\end{cases}$$

and

$$f(x)=f(x+2\pi).$$

(i) Sketch the graph of the function for $-3\pi<x<3\pi$.

(ii) Find the Fourier coefficients corresponding to the function.

(iii) Show that the Fourier series is given by

$$f(x)=\frac{8}{\pi}\sum_{n=1}^{\infty}\frac{n\sin 2nx}{(2n-1)(2n+1)}.$$

(iv) By choosing an appropriate value for x, deduce the sum of the infinite series

$$\frac{1}{1\cdot 3}-\frac{3}{5\cdot 7}+\frac{5}{9.11}-\cdots.$$

(14 marks)

(b) Find Fourier transform of the following function

(i) $f(t)=3t^3e^{-4t}H(t)$..

(ii) $f(t)=e^{-2it}\delta(t-2)$.

(6 marks)

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Laplace Transforms

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
$t^n,\ n=1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
e^{at}	$\frac{1}{s-a}$	$\delta(t-a)$	e^{-as}
$\sin at$	$\frac{a}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\int_0^t f(u)g(t-u)du$	$F(s).G(s)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$y(t)$	$Y(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$\dot{y}(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$\ddot{y}(t)$	$s^2Y(s)-sy(0)-\dot{y}(0)$
$t^n f(t),\ n=1, 2, 3, \ldots$	$(-1)^n \frac{d^n}{ds^n}[F(s)]$	$\mathcal{L}\{f(t)\}=\frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt, \quad s>0.$	

Fourier Transform: $F\{f(t)\} = \int_{-\infty}^{\infty} e^{-iwt} f(t)dt$

Fourier Series: $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$, where $-\pi < x < \pi$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx;\ a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\ dx \text{ and } b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\ dx$$

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FIRST ORDER LINEAR DIFFERENTIAL EQUATION: $ay' + by = f(x)$

Linear: $\frac{dy}{dx} + P(x)y = Q(x)$ and I. F = $e^{\int P(x)dx}$; $y\, e^{\int P(x)dx} = \int e^{\int P(x)dx} Q(x)\,dx + C$

Exact: $M(x,y)dx + N(x,y)dy = 0$ and $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, then $u(x,y) = \int M(x,y)dx + g(y)$

SECOND ORDER LINEAR DIFFERENTIAL EQUATION: $ay'' + by' + cy = f(x)$

Homogeneous Solution:

Type of roots	Complementary Function
Roots are real and distinct $(m_1 \neq m_2)$	$y_c = Ae^{m_1 x} + Be^{m_2 x}$
Roots are real and equal $(m_1 = m_2)$	$y_c = (A + xB)e^{mx}$
Roots are imaginary $(m = \alpha \pm i\beta)$	$y_c = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Method of undetermined coefficients:

$f(x)$	$y_p(x)$
$P_n(x)e^{\lambda x}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)\, e^{\lambda x}$
$P_n(x)\begin{cases}\cos\omega x \\ \sin\omega x\end{cases}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)\cos\omega x$ $x^r(C_n x^n + C_{n-1}x^{n-1} + \cdots + C_1 x + C_0)\sin\omega x$
$Ce^{\lambda x}\begin{cases}\cos\omega x \\ \sin\omega x\end{cases}$	$x^r\, e^{\lambda x}\, K\cos\omega x + L\sin\omega x)$
$P_n(x)e^{\lambda x}\begin{cases}\cos\omega x \\ \sin\omega x\end{cases}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)e^{\lambda x}\cos\omega x$ $x^r(C_n x^n + C_{n-1}x^{n-1} + \cdots + C_1 x + C_0)e^{\lambda x}\sin\omega x$

Variation of Parameters:

$y_c = y_1 + y_2$; $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$; $u = -\int \frac{y_2 f(x)}{aW}dx + A$ and $v = \int \frac{y_1 f(x)}{aW}dx + B$

General Solution $y = uy_1 + vy_2$

SYSTEM OF FIRST ORDER LINEAR DIFFERENTIAL EQUATION

Eigen value and Eigen vector: $|A - \lambda I| = 0$ and $\varphi_1 = \begin{bmatrix}\varphi_{11} \\ \varphi_{12}\end{bmatrix}$, $\varphi_2 = \begin{bmatrix}\varphi_{21} \\ \varphi_{22}\end{bmatrix}$